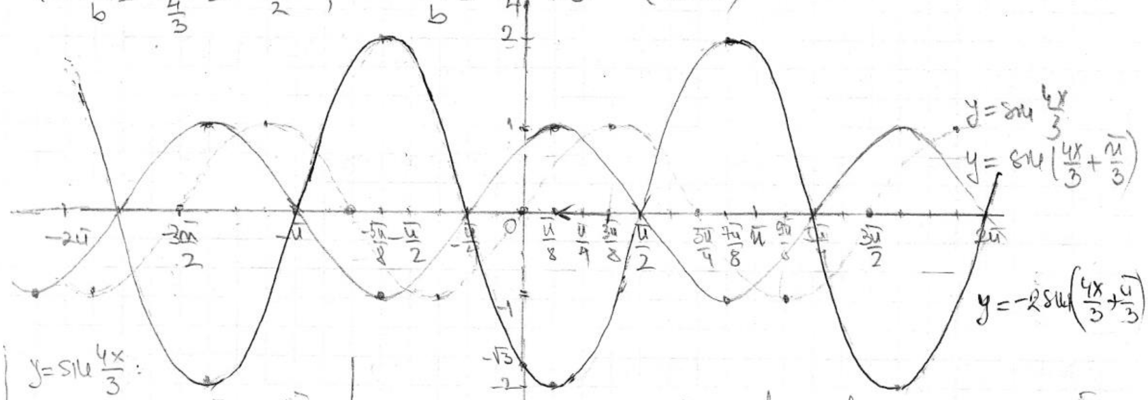
1 ИСПИТАТИ ОСОБИНЕ И СКИЦИРАТИ ГРАФИК Ф-ЈЕ:

I група $y=-2\sin\left(\frac{4x}{3}+\frac{\pi}{3}\right)$

$T=\frac{2\pi}{b}=\frac{2\pi}{\frac{4}{3}}=\frac{3\pi}{2}$; $\frac{c}{b}=\frac{\pi}{4}>0$ $(\longleftarrow)$



$y=\sin\frac{4x}{3}$:

$y=0$: $x=0$, $x=\frac{3\pi}{4}$, $x=\frac{3\pi}{4}$

$y_{max}=1$: $\frac{4x}{3}=\frac{\pi}{2}\Rightarrow x=\frac{3\pi}{8}$

$y_{min}=-1$: $\frac{4x}{3}=\frac{3\pi}{2}\Rightarrow x=\frac{9\pi}{8}$

1) Домен: $x\in(-\infty,+\infty)$

2) Парност: ни парна, ни непарна

3) Периодичност: $T=\frac{3\pi}{2}$

4) Ограниченост: $-2\le y\le 2$

5) Нуле: $x=-\frac{\pi}{4}+\frac{3k\pi}{4}$, $k\in Z$; $x=0$: $y=-\sqrt{3}$.

6) Екстремуми: $y_{max}=2$: $x=-\frac{5\pi}{8}+\frac{3k\pi}{2}$ $k\in Z$

$y_{min}=-2$: $x=\frac{\pi}{8}+\frac{3k\pi}{2}$

7) Знак: $y>0$: $x\in\left(-\pi+\frac{3k\pi}{2}, -\frac{\pi}{4}+\frac{3k\pi}{2}\right)$ $k\in Z$

$y<0$: $x\in\left(-\frac{\pi}{4}+\frac{3k\pi}{2}, \frac{\pi}{2}+\frac{3k\pi}{2}\right)$

8) Монотоност: $y\nearrow$: $x\in\left[\frac{\pi}{8}+\frac{3k\pi}{2}, \frac{7\pi}{8}+\frac{3k\pi}{2}\right]$ $k\in Z$

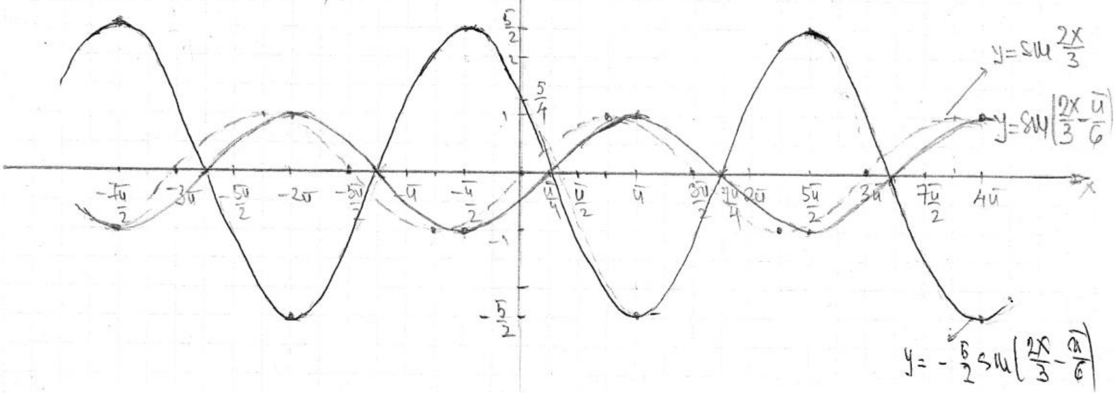
$y\searrow$: $x\in\left[-\frac{5\pi}{8}+\frac{3k\pi}{2}, \frac{\pi}{8}+\frac{3k\pi}{2}\right]$

II група $y=-\frac{5}{2}\cdot\sin\left(\frac{2x}{3}-\frac{\pi}{6}\right)$ $T=3\pi$, $\frac{c}{b}=-\frac{\pi}{4}<0$ $(\longrightarrow)$

$y=\sin\frac{2x}{3}$

$y=0$: $x=0$, $x=\frac{3\pi}{2}$, $x=3\pi$

$y_{max}=1$: $x=\frac{3\pi}{4}$; $y_{min}=-1$: $x=\frac{9\pi}{4}$



1) Домен: $x \in (-\infty, +\infty)$

2) Парност: ни парне, ни непарне

3) Периодичност: $T = 3\pi$

4) Ограниченост: $-\frac{5}{2} \le y \le \frac{5}{2}$

5) Нуле: $y=0$: $x = \frac{\pi}{4} + \frac{3k\pi}{2}, k \in Z$

$x=0$: $y = \frac{5}{4}$.

6) Екстремуми:

$y_{max} = \frac{5}{2}$: $x = -\frac{\pi}{2} + 3k\pi$, $k \in Z$

$y_{min} = -\frac{5}{2}$: $x = \pi + 3k\pi$,

7) Знак: $y > 0$: $x \in \left(-\frac{5\pi}{4} + 3k\pi, \frac{\pi}{4} + 3k\pi\right)$ $k \in Z$

$y < 0$: $x \in \left(\frac{\pi}{4} + 3k\pi, \frac{7\pi}{4} + 3k\pi\right)$

7) Монотоност: $y \nearrow$: $x \in \left[\pi + 3k\pi, \frac{5\pi}{2} + 3k\pi\right]$ $k \in Z$

$y \searrow$: $x \in \left[-\frac{\pi}{2} + 3k\pi, \pi + 3k\pi\right]$

$\boxed{2}$ I група Израчунати $\cos\left(\frac{\pi}{2} - \alpha - \beta\right)$ ако је $\sin\alpha = -\frac{3}{5}$, $\cos\beta = -\frac{5}{13}$

$\frac{3\pi}{2} < \alpha < 2\pi$, $\frac{\pi}{2} < \beta < \pi$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{4}{5}$

$\sin^2\beta + \cos^2\beta = 1 \Rightarrow \sin\beta = \frac{12}{13}$

$\cos\left(\frac{\pi}{2} - \alpha - \beta\right) = \cos\left(\frac{\pi}{2} - (\alpha+\beta)\right)$

$= \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= -\frac{3}{5}\cdot\left(-\frac{5}{13}\right) + \frac{4}{5}\cdot\frac{12}{13} = \frac{63}{65}$

II група Израчунати $\cos(\pi - \alpha + \beta)$ ако је $\cos\alpha = \frac{4}{5}$, $\sin\beta = -\frac{24}{25}$

и $\frac{3\pi}{2} < \alpha < 2\pi$, $\pi < \beta < \frac{3\pi}{2}$.

$\sin\alpha = -\frac{3}{5}$; $\cos\beta = -\frac{7}{25}$

$\cos(\pi - \alpha + \beta) = \cos(\pi - (\alpha - \beta)) =$

$= -\sin(\alpha - \beta) = -\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{3}{5}\cdot\left(-\frac{7}{25}\right) - \frac{96}{125} = -\frac{117}{125}$

$\boxed{3}$ Доказати идентитет: $\frac{2 - \sin 4\alpha \cdot ctg 2\alpha}{\sin 4\alpha} = tg 2\alpha$

$$L = \frac{2 - 2\cancel{\sin 2\alpha}\cdot\cos 2\alpha\cdot\frac{\cos 2\alpha}{\cancel{\sin 2\alpha}}}{2\sin 2\alpha\cos 2\alpha} = \frac{2(1-\cos^2 2\alpha)}{2\sin 2\alpha\cos 2\alpha} = \frac{\cancel{2}\sin^2 2\alpha}{\cancel{2}\sin 2\alpha\cdot\cos 2\alpha} = tg 2\alpha = D$$

II група $\frac{\cos 2\alpha}{1+\cos 2\alpha} : \frac{1+\cos 4\alpha}{\sin 4\alpha} = tg\alpha$

$$L = \frac{\cos 2\alpha}{1+\cos 2\alpha}\cdot\frac{2\sin 2\alpha\cdot\cos 2\alpha}{1+\cos^2 2\alpha - \sin^2 2\alpha} = \frac{\cancel{\cos 2\alpha}}{1+\cos 2\alpha}\cdot\frac{\cancel{2}\sin 2\alpha\,\cancel{\cos 2\alpha}}{\cancel{2}\cancel{\cos^2 2\alpha}} =$$

$$= \frac{\sin 2\alpha}{1+\cos 2\alpha} = \frac{2\sin\alpha\cos\alpha}{1+\cos^2\alpha - \sin^2\alpha} = \frac{\cancel{2}\sin\alpha\,\cancel{\cos\alpha}}{\cancel{2}\cos^{\cancel{2}}\alpha} = tg\alpha = D$$

$\boxed{4}$ Доказати да је:

I група $\frac{1 - tg^2\frac{\pi}{8}}{1 + tg^2\frac{\pi}{8}} = \frac{\sqrt{2}}{2}$

$$tg^2\frac{\pi}{8} = \frac{1 - \cos\frac{\pi}{4}}{1 + \cos\frac{\pi}{4}} = \frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{2+\sqrt{2}} = \frac{(2-\sqrt{2})^2}{2} = \frac{4 - 4\sqrt{2} + 2}{2} = \frac{6 - 4\sqrt{2}}{2} = 3 - 2\sqrt{2}$$

$$L = \frac{1-tg^2\frac{\pi}{8}}{1+tg^2\frac{\pi}{8}} = \frac{1-3+2\sqrt{2}}{1+3-2\sqrt{2}} = \frac{-2+2\sqrt{2}}{4-2\sqrt{2}} = \frac{-1+\sqrt{2}}{2-\sqrt{2}} = \frac{\cancel{\sqrt{2}-1}}{\sqrt{2}(\cancel{\sqrt{2}-1})} = \frac{\sqrt{2}}{2} = D$$ ③

II група

$$\frac{1-tg^2\frac{\pi}{12}}{1+tg^2\frac{\pi}{12}} = \frac{\sqrt{3}}{2}$$

$$tg^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}} = \frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2-\sqrt{3})^2}{4-3} = 4-4\sqrt{3}+3 = 7-4\sqrt{3}$$

$$L = \frac{1-tg^2\frac{\pi}{12}}{1+tg^2\frac{\pi}{12}} = \frac{1-7+4\sqrt{3}}{1+7-4\sqrt{3}} = \frac{-6+4\sqrt{3}}{8-4\sqrt{3}} = \frac{2(-3+2\sqrt{3})}{{}_2 4(2-\sqrt{3})} = \frac{\sqrt{3}(2-\sqrt{3})}{2(2-\sqrt{3})} = \frac{\sqrt{3}}{2} = D$$

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5 Израчунати

I група $\sin 3x$ у функцији од $\sin x$

$$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x = 2\sin x\cos^2 x + (\cos^2 x - \sin^2 x)\sin x$$
$$= 3\sin x(1-\sin^2 x) - \sin^3 x = 3\sin x - 4\sin^3 x.$$

II група $\cos 3x$ у ф-ји од $\cos x$

$$\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x\cdot\sin x =$$
$$= (\cos^2 x - \sin^2 x)\cos x - 2\sin x\cos x\cdot\sin x = \cos^3 x - 3\sin^2 x\cos x =$$
$$= \cos^3 x - 3\cos x + 3\cos^3 x = 4\cos^3 x - 3\cos x.$$