① Решити детерминанту:

$$(\mathrm{I})\ \begin{vmatrix}34&92&14\\20&49&8\\46&104&24\end{vmatrix} = 4\cdot\begin{vmatrix}17&46&7\\20&49&8\\23&52&12\end{vmatrix}\begin{matrix}(-1)\\ +\\ \ \end{matrix} =$$

$$= 4\cdot\begin{vmatrix}17&46&7\\3&3&1\\6&6&5\end{vmatrix}\begin{matrix}\ \\ (-2)\\ \ \end{matrix} = 4\cdot\begin{vmatrix}17&46&7\\3&3&1\\0&0&3\end{vmatrix} =$$

$$= 12\cdot\begin{vmatrix}17&46\\3&3\end{vmatrix} = 36\cdot\begin{vmatrix}17&46\\1&1\end{vmatrix} = 36\cdot(-29) = -1044.$$

$$(\mathrm{II})\ \begin{vmatrix}24&27&30\\52&65&78\\91&92&94\end{vmatrix} = 3\cdot\begin{vmatrix}8&9&10\\52&65&78\\91&92&94\end{vmatrix} = 3\cdot13\cdot\begin{vmatrix}8&9&10\\4&5&6\\91&92&94\end{vmatrix} =$$

(-1)

$$= 39\cdot\begin{vmatrix}8&1&2\\4&1&2\\91&1&3\end{vmatrix} = 39\cdot\begin{vmatrix}8&1&0\\4&1&0\\91&1&1\end{vmatrix} = 39\begin{vmatrix}8&1\\4&1\end{vmatrix} =$$

(-2)

$$= 39\cdot4\cdot\begin{vmatrix}2&1\\1&1\end{vmatrix} = 156.$$

② Решити једначину:

$$\mathrm{I}\ \begin{vmatrix}-2&8&x+3\\1&6&3\\-1&2x&x\end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix}-2&8&x+3\\1&6&3\\-1&2x&x\end{vmatrix} /\cdot2\ /\cdot1 = 2\cdot\begin{vmatrix}0&10&x+9\\1&3&3\\0&x+3&x+3\end{vmatrix} =$$

$$= -2 \cdot \begin{vmatrix} 10 & x+9 \\ x+3 & x+3 \end{vmatrix} = -2(x+3)\begin{vmatrix} 10 & x+9 \\ 1 & 1 \end{vmatrix} = -2(x+3)(1-x)$$

Како је $\Delta = 0$: $-2(x+3)(1-x) = 0 \Rightarrow x=-3 \lor x=1$

(II) $\begin{vmatrix} x-2 & 1 & -1 \\ -1 & x+2 & 1 \\ -1 & x & -1 \end{vmatrix} = 0$

$$\Delta = \begin{vmatrix} x-2 & 1 & -1 \\ -1 & x+2 & +1 \\ -1 & x & -1 \end{vmatrix} = \begin{vmatrix} x-3 & x+3 & 0 \\ -1 & x+2 & 1 \\ -2 & 2x+2 & 0 \end{vmatrix} =$$

$$= -1 \cdot \begin{vmatrix} x-3 & x+3 \\ -2 & 2(x+1) \end{vmatrix} = -2\begin{vmatrix} x-3 & x+3 \\ -1 & x+1 \end{vmatrix} = -2(x^2-2x-\cancel{3}+x+\cancel{3})$$

$$= -2(x^2-x) = -2 \cdot x(x-1)$$

Како је $\Delta = 0$: $x=0 \lor x=1$.

(3) Крамеровим правилом решити систем.

$$\text{(I)} \quad \begin{aligned} 2x + 3y - z &= 5 \\ x + y + 2z &= 7 \\ 2x - y + z &= 1 \end{aligned}$$

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 8 & 3 & 2 \\ 3 & 1 & 3 \\ 0 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 8 & 2 \\ 3 & 3 \end{vmatrix} = 3 \cdot 6 = 18$$

$$D_x = \begin{vmatrix} 5 & 3 & -1 \\ 7 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 8 & 3 & 2 \\ 8 & 1 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 8 \cdot \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 8$$

$$D_y = \begin{vmatrix} 2 & 5 & -1 \\ 1 & 7 & 2 \\ 2 & 1 & 1 \end{vmatrix} \begin{matrix} /\cdot 2 \\ + \\ + \end{matrix} = \begin{vmatrix} 2 & 5 & -1 \\ 5 & 17 & 0 \\ 4 & 6 & 0 \end{vmatrix} = -(30-68) = 38$$

$$D_z = \begin{vmatrix} 2 & 3 & 5 \\ 1 & 1 & 7 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 8 & 3 & 8 \\ 3 & 1 & 8 \\ 0 & -1 & 0 \end{vmatrix} = 8 \cdot \begin{vmatrix} 8 & 1 \\ 3 & 1 \end{vmatrix} = 40$$

$$x = \frac{D_x}{D} = \frac{8}{18} = \frac{4}{9}, \quad y = \frac{38}{18} = \frac{19}{9}, \quad z = \frac{40}{18} = \frac{20}{9}$$

$$(x, y, z) = \left(\frac{4}{9}, \frac{19}{9}, \frac{20}{9}\right)$$

II

$$\begin{aligned} 2x - y + z &= 7 \\ 3x + y - 5z &= 13 \\ x + y + z &= 5 \end{aligned}$$

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 1 \\ 5 & 0 & -4 \\ 3 & 0 & 2 \end{vmatrix} = \cancel{32}\ 22$$

$$D_x = \begin{vmatrix} 7 & -1 & 1 \\ 13 & 1 & -5 \\ 5 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 7 & -1 & 3 \\ 20 & 0 & -4 \\ 12 & 0 & 2 \end{vmatrix} = \cancel{128}\ 88$$

$$D_y = \begin{vmatrix} 2 & 7 & 1 \\ 3 & 13 & -5 \\ 1 & 5 & 1 \end{vmatrix} (-2)/(-3) = \begin{vmatrix} 0 & -3 & -1 \\ 0 & -2 & -8 \\ 1 & 5 & 1 \end{vmatrix} = 22$$

$$D_z = \begin{vmatrix} 2 & -1 & 7 \\ 3 & 1 & 13 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 7 \\ 5 & 0 & 20 \\ 3 & 0 & 12 \end{vmatrix} = 0$$

$$x = \frac{88}{22} = 4 \quad y = \frac{22}{22} = \cancel{\frac{14}{16}}\ 1 \quad z = 0 \quad (x, y, z) = (4, 1, 0)$$

(4) Доказати:

I) $$\begin{vmatrix} 2x+y+z & y & z \\ x & x+2y+z & z \\ x & y & x+y+2z \end{vmatrix} = 2(x+y+z)^3$$

$$L = \begin{vmatrix} 2x+y+z & y & z \\ x & x+2y+z & z \\ x & y & x+y+2z \end{vmatrix} (-1) =$$

$$= \begin{vmatrix} x+y+z & 0 & -x-y-z \\ 0 & x+y+z & -x-y-z \\ x & y & x+y+2z \end{vmatrix} =$$

$$= (x+y+z)^2 \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ x & y & x+y+2z \end{vmatrix} =$$

$$= (x+y+z)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ x & y & 2x+y+2z \end{vmatrix} = (x+y+z)^2 (2x+2y+2z) =$$

$$= 2(x+y+z)^3 = D.$$

(II) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$L \equiv \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} =$$

(-1)

$$= \begin{vmatrix} -a-b-c & 0 & 2a \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix} = (a+b+c)^2 \begin{vmatrix} -1 & 0 & 2a \\ 0 & -1 & 2b \\ 1 & 1 & c-a-b \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} 0 & 1 & c+a-b \\ 0 & -1 & 2b \\ 1 & 1 & c-a-b \end{vmatrix} = (a+b+c)^2 \begin{vmatrix} 1 & c+a-b \\ -1 & 2b \end{vmatrix}$$

$$= (a+b+c)^2 (2b+c+a-b) = (a+b+c)^3 \equiv D$$