① Решити једначину:

$\sin 2x + 2\cos^2 x = 0$

$2\sin x \cos x + 2\cos^2 x = 0$

$2\cos x(\sin x + \cos x) = 0$

$2\cos x \cdot \cos x\left(\frac{\sin x}{\cos x} + 1\right) = 0$

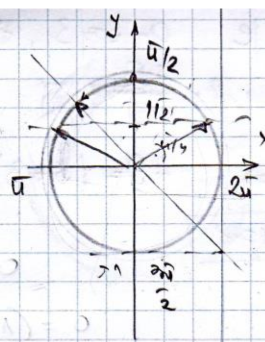
$2\cos^2 x \cdot (\mathrm{tg}\, x + 1) = 0$

$\cos x = 0 \lor \mathrm{tg}\, x + 1 = 0$

$\boxed{x = \frac{\pi}{2} + k\pi} \lor \mathrm{tg}\, x = -1$

$\mathrm{tg}\, x = \mathrm{tg}\left(\frac{3\pi}{4} + k\pi\right)$

$\boxed{x_2 = \frac{3\pi}{4} + l\pi}, \; k, l \in Z$



$\sin\frac{x}{2} + \cos x = 1$

$\sin\frac{x}{2} = 1 - \cos x$

$\sin\frac{x}{2} = 2 \cdot \sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} - \sin\frac{x}{2} = 0$

$\sin\frac{x}{2}\left(2\sin\frac{x}{2} - 1\right) = 0$

$\sin\frac{x}{2} = 0 \lor \sin\frac{x}{2} = \frac{1}{2}$

$\frac{x}{2} = k\pi \quad \Rightarrow \boxed{x_1 = 2k\pi}$

$\frac{x}{2} = \frac{\pi}{6} + 2p\pi \lor \frac{x}{2} = \frac{5\pi}{6} + 2l\pi$

$\boxed{x = \frac{\pi}{3} + 4p\pi} \lor \boxed{x = \frac{5\pi}{3} + 4l\pi}$

$k, l, p \in Z$

② Одредити остале основне елементе троугла ако је $\alpha = 45^\circ$, $\beta = 60^\circ$ и полупречник описаног круга $r_0 = 2\sqrt{6}$ cm

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r_0$

$\frac{a}{\sin 45^\circ} = 4\sqrt{6} \Rightarrow a = \frac{\sqrt{2}}{2} \cdot 4\sqrt{6} = 4\sqrt{3}$ cm

$\frac{b}{\sin 60} = 4\sqrt{6} \Rightarrow b = \frac{\sqrt{3}}{2} \cdot 4\sqrt{6} = 6\sqrt{2}$ cm

$\frac{c}{\sin\gamma} = 4\sqrt{6} \Rightarrow c = \frac{4\sqrt{6} \cdot \sqrt{2}}{4}(1+\sqrt{3}) = 2\sqrt{3}(1+\sqrt{3})$ cm

$\gamma = 75^\circ$

$\sin 75^\circ = \sin(30^\circ + 45^\circ) =$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} =$

$= \frac{\sqrt{2}}{4}(1+\sqrt{3})$

Одредити остале елементе троугла ако је $c = 3\sqrt{2}$, $\alpha = 60^\circ$ $\beta = 75^\circ$.

$\gamma = 45^\circ \Rightarrow \frac{c}{\sin\gamma} = \frac{3\sqrt{2}}{\sin 45^\circ} = \frac{3\sqrt{2}}{\frac{\sqrt{2}}{2}} = 6$

$\frac{a}{\sin\alpha} = 6 \Rightarrow a = \sin 60^\circ \cdot 6 = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$ cm

$\frac{b}{\sin\beta} = 6 \Rightarrow b = 6 \cdot \sin 75^\circ = 6 \cdot \frac{\sqrt{2}}{4}(1+\sqrt{3}) = \frac{3\sqrt{2}}{2}(1+\sqrt{3})$ cm

③ Одреди странице a и b ако је $c = 2$, $a : b = \sqrt{7} : 3$, $\alpha = 60^\circ$

$a = \frac{b\sqrt{7}}{3}$; $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\frac{7b^2}{9} = b^2 + 4 - 2 \cdot b \cdot 2 \cdot \frac{1}{2} \Rightarrow 2b^2 - 18b + 36 = 0 \; /:2$

$\boxed{b = 3:\ a = \sqrt{7}\text{ cm}}$; $\boxed{b = 6:\ a = 2\sqrt{7}\text{ cm}}$

Одредити a и b ако је $a-b=3$, $\gamma=60^\circ$ $r_0=\frac{7\sqrt{3}}{3}$.

$\frac{c}{\sin\gamma}=2r_0 \Rightarrow c=\frac{\sqrt{3}}{2}\cdot 2\cdot\frac{7\sqrt{3}}{3}$ $\boxed{c=7\,cm}$

$c^2=a^2+b^2-2ab\cos\gamma \Rightarrow 49=(b+3)^2+b^2-2b(b+3)\cdot\frac{1}{2}$

$b^2+3b-40=0 \Rightarrow \boxed{b=5;\ a=8}$

(4) Решити једначину:

$3^{x-1}+3^{x-2}+3^{x-3}+3^{x-4}+3^{x-5}+3^{x-6}=364$

$3^x\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\frac{1}{3^6}\right)=364$

$3^x\cdot\frac{3^5+3^4+3^3+3^2+3+1}{3^6}=364$

$3^x\cdot\frac{243+81+27+9+3+1}{3^6}=364$

$3^x=\frac{\cancel{364}\cdot 3^{6}}{\cancel{364}} \Rightarrow \boxed{x=6}$

Решити једначину:

$2^{4x}+2^{4x-1}+4^{2x-1}+2^{4x-3}+16^{x-1}=31$

$2^{4x}+2^{4x}\cdot 2^{-1}+2^{4x}\cdot 2^{-2}+2^{4x}\cdot 2^{-3}+2^{4x}\cdot 2^{-4}=31$

$2^{4x}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)=31$

$2^{4x}\,\frac{16+8+4+2+1}{16}=31$

$2^{4x}=\frac{\cancel{31}\cdot 16}{\cancel{31}} \Rightarrow 4x=4$

$\boxed{x=1}$