решења:

(1) $9x+y, \_, \_, \_, \_, \_, \_, \_, x+9y$

$a_1 = 9x+y$, $a_9 = x+9y$; $a_9 = a_1 + 8d$; $x+9y = 9x+y+8d$; $8y-8x = 8d$; $\boxed{d = y-x}$

Низ:

$(9x+y, 8x+2y, 7x+3y, 6x+4y, 5x+5y, 4x+6y, 3x+7y, 2x+8y, x+9y)$

или по обрасцу: $d = \frac{b-a}{r+1}$, где је r – бр. чланова које треба уметнути између чланова a, b.

$$d = \frac{x+9y-(9x+y)}{7+1} = \frac{8y-8x}{8} = y-x.$$

(2) $x+y+z=21$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{7}{12}$; $y = x\cdot q$, $z = x\cdot q^2$; $x + x\cdot q + x\cdot q^2 = 21$, $\frac{1}{x} + \frac{1}{x\cdot q} + \frac{1}{x\cdot q^2} = \frac{7}{12}$

$x(1+q+q^2) = 21$, $\frac{q^2+q+1}{xq^2} = \frac{7}{12}$; $\boxed{x = \frac{12(1+q+q^2)}{7q^2}}$

$\frac{^4\cancel{12}(1+q+q^2)}{\cancel{7}q^2}\cdot(1+q+q^2) = \cancel{21}^{\,7}$

$4(1+q+q^2)^2 = 7^2q^2 \quad \Rightarrow \quad 2(1+q+q^2) = \pm 7q$

I: $2+2q+2q^2 = 7q$

$2q^2-5q+2 = 0$

$q_{1,2} = \frac{5\pm 3}{4} = 2$ или $\frac{1}{2}$

II: $2+2q+2q^2 = -7q$

$2q^2+9q+2=0$

$q_{1,2} = \frac{-9\pm\sqrt{65}}{4} = \frac{-9+\sqrt{65}}{4}$ или $\frac{-9-\sqrt{65}}{4}$

$\Downarrow$

Није растући низ.

Растући низ: $q=2$

$x = \frac{12\cdot 7}{7\cdot 4} = 3$

$(3, 6, 12)$

(3) $a_1,\ a_2,\ a_3,\ a_4$ аритметички низ

$a_1-2,\ a_2-7,\ a_3-9,\ a_4-5$ геометријски

$$\begin{cases}(a_2-7)^2=(a_1-2)(a_3-9)\\ (a_3-9)^2=(a_2-7)(a_4-5)\end{cases}\qquad \begin{cases}a_2^2-14a_2+49=a_1a_3-9a_1-2a_3+18\\ a_3^2-18a_3+81=a_2\cdot a_4-5\cdot a_2-7a_4+35\end{cases}$$

$a_2=a_1+d,\ a_3=a_1+2d,\ a_4=a_1+3d$ јер је низ аритмет.

па је:

$$(a_1+d)^2-14(a_1+d)+49=a_1(a_1+2d)-9a_1-2(a_1+2d)+18$$
$$(a_1+2d)^2-18(a_1+2d)+81=(a_1+d)(a_1+3d)-5(a_1+d)-7(a_1+3d)+35$$

$$\cancel{a_1^2}+\cancel{2a_1d}+d^2-14a_1-\underline{14d}+49=\cancel{a_1^2}+\cancel{2a_1d}-9a_1-2a_1-\underline{4d}+18$$
$$\cancel{a_1^2}+\cancel{4a_1d}+4d^2-18a_1-\underline{36d}+81=\cancel{a_1^2}+\cancel{4a_1d}+3d^2-5a_1-\underline{5d}-7a_1-\underline{21d}+35$$

$$d^2-10d-3a_1+31=0 \quad /\cdot(-1)$$
$$d^2-10d-6a_1+46=0$$

$$-3a_1=-15 \Rightarrow \boxed{a_1=5} \qquad d^2-10d+16=0$$

$$d_{1,2}=\frac{10\pm 6}{2}=\begin{cases}8\\2\end{cases}$$

Бр. су: 5, 13, 21, 29 или 5, 7, 9, 11

(геом. низ) 3, 6, 12, 24   3, 0, 0, 6

(4) $$\lim_{n\to\infty}\frac{((2n+1)^2)^2-((n-1)^2)^2}{((2n+1)^2)^2+((n-1)^2)^2}=\lim_{n\to\infty}\frac{(4n^2+4n+1)^2-(n^2-2n+1)^2}{(4n^2+4n+1)^2+(n^2-2n+1)^2}=$$

$$=\lim_{n\to\infty}\frac{16n^4+32n^3+16n^2+8n^2+8n+\cancel{1}-n^4+4n^3-4n^2-2n^2+4n-\cancel{1}}{16n^4+32n^3+16n^2+8n^2+8n+1+n^4-4n^3+4n^2+2n^2-4n+1}=$$

$$=\lim_{n\to\infty}\frac{15n^4+36n^3+18n^2+12n}{17n^4+28n^3+30n^2+4n+2}=\lim_{n\to\infty}\frac{15+\cancel{\frac{36}{n}}+\cancel{\frac{18}{n^2}}+\cancel{\frac{12}{n^3}}}{17+\cancel{\frac{28}{n}}+\cancel{\frac{30}{n^2}}+\cancel{\frac{4}{n^3}}+\cancel{\frac{2}{n^4}}}=$$

$$=\frac{15}{17}$$

(5) $$z=-2+2\sqrt{3}\ \overset{*}{\Rightarrow}\ \boxed{z=4\left(\cos\frac{2\pi}{3}+i\cdot\sin\frac{2\pi}{3}\right)}$$

$$*\left(r=\sqrt{4+4\cdot 3}=4 \quad \begin{cases}-2=4\cos\varphi\\ 2\sqrt{3}=4\cdot\sin\varphi\end{cases}\ \begin{cases}\cos\varphi=-\frac{1}{2}\\ \sin\varphi=\frac{\sqrt{3}}{2}\end{cases}\ \varphi\in II \Rightarrow \boxed{\varphi=\frac{2\pi}{3}}\right)$$

$w = 2-2i \overset{**}{\Rightarrow} \boxed{w = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\cdot\sin\left(-\frac{\pi}{4}\right)\right)}$

$** \left(r = \sqrt{4+4} = 2\sqrt{2} \;;\; \left.\begin{array}{l} 2 = 2\sqrt{2}\cos\varphi \\ -2 = 2\sqrt{2}\sin\varphi \end{array}\right\} \left.\begin{array}{l} \cos\varphi = \frac{\sqrt{2}}{2} \\ \sin\varphi = -\frac{\sqrt{2}}{2} \end{array}\right\} \theta \in IV \;;\; \theta = -\frac{\pi}{4} \right)$

a) $z\cdot w = 4\cdot 2\sqrt{2}\left(\cos\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) + i\sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)\right)$

$= 8\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

б) $\frac{w}{z} = \frac{2\sqrt{2}}{4}\left(\cos\left(-\frac{\pi}{4} - \frac{2\pi}{3}\right) + i\sin\left(-\frac{\pi}{4} - \frac{2\pi}{3}\right)\right)$

$= \frac{\sqrt{2}}{2}\left(\cos\left(-\frac{11\pi}{12}\right) + i\sin\left(-\frac{11\pi}{12}\right)\right)$

в) $z^3 = 4^3\cdot\left(\cos\frac{3\cdot 2\pi}{3} + i\sin\frac{2\pi\cdot 3}{3}\right) = 64\cdot\left(\cos 2\pi + i\sin 2\pi\right) = 64$

г) $\sqrt[3]{w} = \sqrt[3]{2\sqrt{2}}\left(\cos\frac{-\frac{\pi}{4} + 2k\pi}{3} + i\sin\frac{-\frac{\pi}{4} + 2k\pi}{3}\right) \quad k = 0, 1, 2$

$\sqrt[3]{2\sqrt{2}} = \sqrt[3]{\sqrt{2^3}} = \sqrt[6]{2^3} = \sqrt{2}$

$w_0 = \sqrt[3]{w} = \sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$

$w_1 = \sqrt[3]{w} = \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

$w_2 = \sqrt[3]{w} = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$