

① Ако је  $\sin x - \cos x = a$ , израчунати  $\cos^2 2x$ .

$$\cos^2 2x = (\cos^2 x - \sin^2 x)^2 = ((\cos x - \sin x)(\cos x + \sin x))^2 = (-a \cdot (\sin x + \cos x))^2$$

$$= a^2 \cdot (\sin^2 x + 2 \sin x \cos x + \cos^2 x) = a^2 (1 + \sin 2x) \quad *$$

\* Из  $\sin x - \cos x = a$  је:  $a^2 = (\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2 \sin x \cos x$

$$a^2 = 1 - \sin 2x \Rightarrow \sin 2x = 1 - a^2 \quad (**)$$

\*  $\Rightarrow \cos^2 2x = a^2 (1 + 1 - a^2) \Rightarrow \cos^2 2x = a^2 (2 - a^2)$ .

② Решити једначину:

(I)  $2 \cos(2x + \frac{\pi}{3}) = \sqrt{2}$

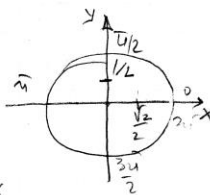
$$\cos(2x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$$

$$2x + \frac{\pi}{3} = \pm \frac{\pi}{4} + 2k\pi \quad 2x$$

$$2x = -\frac{7\pi}{12} + 2k\pi$$

$$x_1 = -\frac{7\pi}{24} + k\pi$$

$$x_2 = -\frac{\pi}{24} + k\pi$$



II  $2 \sin(3x - \frac{\pi}{3}) = 1$

$$\sin(3x - \frac{\pi}{3}) = \frac{1}{2}$$

1)  $3x - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi$

$$3x = \frac{\pi}{2} + 2k\pi \quad x = \frac{\pi}{6} + \frac{2k\pi}{3}$$

2)  $3x - \frac{\pi}{3} = \frac{5\pi}{6} + 2k\pi$

$$3x = \frac{7\pi}{6} + 2k\pi$$

$$x_2 = \frac{7\pi}{18} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

③ I Определити остале елементе троугла ако је  $\alpha = 60^\circ$ ,  $\beta = 75^\circ$ ,  $c = 3\sqrt{2}$ .

$$\frac{c}{\sin \gamma} = \frac{3\sqrt{2}}{\frac{\sqrt{2}}{2}} = 6 \Rightarrow \frac{a}{\sin \alpha} = 6 \Rightarrow a = 6 \cdot \sin 60^\circ = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ cm}$$

$$\frac{b}{\sin \beta} = 6 \Rightarrow b = 6 \cdot \sin 75^\circ = 6 \cdot \frac{\sqrt{2}}{4} (1 + \sqrt{3}) = \frac{3\sqrt{2}(1 + \sqrt{3})}{2}$$

II Определити остале елементе троугла ако је  $\alpha = 45^\circ$ ,  $\beta = 60^\circ$  и полупречник описаног круга је  $r_0 = 2\sqrt{6}$  cm.

$$\frac{a}{\sin \alpha} = 2r_0 \Rightarrow a = \frac{\sqrt{2}}{2} \cdot 4\sqrt{6} = 2\sqrt{12} \Rightarrow |a = 4\sqrt{3} \text{ cm}, \quad \gamma = 75^\circ$$

$$\frac{b}{\sin \beta} = 2r_0 \Rightarrow b = 2 \cdot 2\sqrt{6} \cdot \frac{\sqrt{3}}{2} = 2\sqrt{18} \Rightarrow |b = 6\sqrt{2} \text{ cm}| \quad \frac{c}{\sin \gamma} = 2r_0$$

$$c = \frac{\sqrt{2}}{4} (1 + \sqrt{3}) \cdot 4\sqrt{6} = 2\sqrt{3}(1 + \sqrt{3}) \Rightarrow |c = 2\sqrt{3}(1 + \sqrt{3}) \text{ cm}|$$

④ I Определити стране  $a$  и  $b$  троугла ако је:

(I)  $a - b = 3$  и  $r_0 = \frac{7\sqrt{3}}{3}$  cm,  $\gamma = 60^\circ$

(II)  $c = 2$ ,  $a : b = \sqrt{7} : 3$ ,  $\alpha = 60^\circ$ .

$$(I) \frac{c}{\sin \gamma} = 2r_0 \Rightarrow c = 8 \sin 60^\circ \cdot 2 \cdot \frac{\sqrt{3}}{3} = \frac{14}{3} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{3} = 7 \text{ cm.} \quad \boxed{c = 7 \text{ cm}}$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \Rightarrow 49 = (3+b)^2 + b^2 - 2(3+b) \cdot b \cdot \frac{1}{2}$$

$$9 + 6b + 2b^2 - 3b - b^2 - 49 = 0 \Rightarrow b^2 + 3b - 40 = 0 \Rightarrow b_{1,2} = \frac{-3 \pm 13}{2} \Rightarrow \underline{b = 5 \text{ cm}}$$

$$\underline{a = 8 \text{ cm}}$$

$$II \quad a : b = \sqrt{7} : 3, \quad c = 2 \quad \angle = 60^\circ$$

$$a = \sqrt{7}k, \quad b = 3k: \quad 7k^2 = 9k^2 + 4 - 4 \cdot 3k \cdot \frac{1}{2}$$

$$2k^2 - 6k + 4 = 0 \quad k_{1,2} = \frac{6 \pm 2}{4} = \begin{matrix} 2 \\ 1 \end{matrix}$$

$$k=2: \quad a = 2\sqrt{7}, \quad b = 6 \text{ cm}; \quad k=1: \quad a = \sqrt{7} \text{ cm}, \quad b = 3 \text{ cm}$$