1) Испитати промене и скицирати график ф-је:

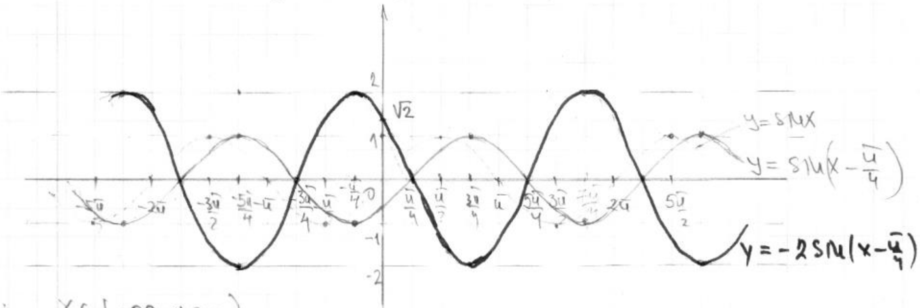
(I група) $y = 2\cdot\cos\left(x+\frac{\pi}{4}\right)$

$y = 2\cos\left(x+\frac{\pi}{4}\right) = 2\cdot\sin\left(\frac{\pi}{2}-\left(x+\frac{\pi}{4}\right)\right) = 2\cdot\sin\left(\frac{\pi}{4}-x\right) = 2\cdot\sin\left(-\left(x-\frac{\pi}{4}\right)\right)$

$y = -2\cdot\sin\left(x-\frac{\pi}{4}\right)$; $a=-2$, $b=1$, $c=-\frac{\pi}{4}$

Основни период: $T = \frac{2\pi}{b} = 2\pi$

Транслација дуж Ox: $\frac{c}{b} = -\frac{\pi}{4} < 0$ (у десну страну за $\frac{\pi}{4}$)



1) Домен: $x\in(-\infty,+\infty)$

2) Парност: ни парна, ни непарна

3) Периодичност: периодична $T=2\pi$

4) Ограниченост: $-2 \le y \le 2$

5) Нуле: $y=0$: $x=\frac{\pi}{4}+k\pi,\ k\in Z$

$x=0$: $y=\sqrt{2}$

6) Екстремне вредности:

$y_{max}=2$: $x=-\frac{\pi}{4}+2k\pi$; $k\in Z$

$y_{min}=-2$: $x=\frac{3\pi}{4}+2k\pi$

7) Знак

$y>0$: $x\in\left(-\frac{3\pi}{4}+2k\pi, \frac{\pi}{4}+2k\pi\right)$ $k\in Z$

$y<0$: $x\in\left(\frac{\pi}{4}+2k\pi, \frac{5\pi}{4}+2k\pi\right)$

8) Монотоност

$y\nearrow$: $x\in\left[-\frac{5\pi}{4}+2k\pi, -\frac{\pi}{4}+2k\pi\right]$

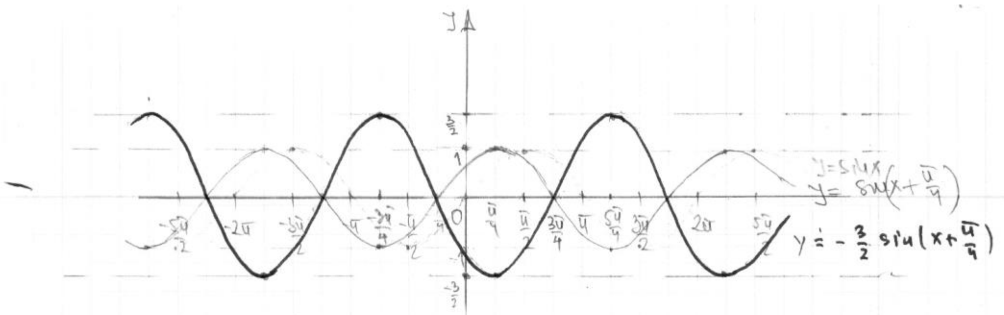
$y\searrow$: $x\in\left[-\frac{\pi}{4}+2k\pi, \frac{3\pi}{4}+2k\pi\right]$, $k\in Z$

(II група) $y = -\frac{3}{2}\cdot\cos\left(\frac{\pi}{4}-x\right)$

$y = -\frac{3}{2}\cdot\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-x\right)\right) = -\frac{3}{2}\sin\left(\frac{\pi}{4}+x\right)$

$y = -\frac{3}{2}\sin\left(x+\frac{\pi}{4}\right)$, $a=-\frac{3}{2}$, $b=1$, $c=\frac{\pi}{4}$

Основни период: $T=2\pi$; транслација: $\frac{c}{b}=\frac{\pi}{4}>0$ (у лево)



1) Домен: $x \in (-\infty, +\infty)$

2) Парност: ни парна, ни непарна

3) Периодичност: $T = 2\pi$

4) Ограниченост: $-\frac{3}{2} \le y \le \frac{3}{2}$

5) Нуле: $y=0$: $x = -\frac{\pi}{4} + k\pi, k \in Z$

$x = 0$: $y = -\frac{3\sqrt{2}}{4}$

6) Екстремне вредности:

$y_{max} = \frac{3}{2}$: $x = -\frac{3\pi}{4} + 2k\pi$

$y_{min} = -\frac{3}{2}$: $x = \frac{\pi}{4} + 2k\pi$ $k \in Z$

7) Знак

$y > 0$: $x \in \left(\frac{3\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi\right)$

$y < 0$: $x \in \left(-\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi\right)$ $k \in Z$

8) Монотоност

$y \nearrow$: $x \in \left[\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi\right]$

$y \searrow$: $x \in \left[-\frac{3\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi\right]$ $k \in Z$

2 (I група) Израчунати $\sin(\alpha+\beta) + \cos(\alpha-\beta)$ ако је:

$\sin\alpha = \frac{3}{5}$, $\cos\beta = -\frac{5}{13}$ и $\frac{\pi}{2} < \alpha < \pi$, $\pi < \beta < \frac{3\pi}{2}$.

$\sin^2\alpha + \cos^2\alpha = 1$ $\qquad$ $\sin^2\beta + \cos^2\beta = 1$

$\cos^2\alpha = \frac{16}{25}$ $\qquad$ $\sin^2\beta \ \frac{144}{169}$

$\cos\alpha = -\frac{4}{5}$ $\qquad$ $\sin\beta = -\frac{12}{13}$

$\sin(\alpha+\beta) + \cos(\alpha-\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta =$

$= \sin\alpha(\sin\beta + \cos\beta) + \cos\alpha(\sin\beta + \cos\beta) =$

$= (\sin\alpha + \cos\alpha)(\sin\beta + \cos\beta) = \left(\frac{3}{5} - \frac{4}{5}\right)\left(-\frac{12}{13} - \frac{5}{13}\right) = -\frac{1}{5}\cdot\left(-\frac{17}{13}\right) = \frac{17}{65}$

(II група) Израчунати $\cos(\alpha+\beta) - \sin(\alpha-\beta)$ ако је:

$\cos\alpha = -\frac{4}{5}$, $\sin\beta = -\frac{24}{25}$, $\pi < \alpha < \frac{3\pi}{2}$, $\frac{3\pi}{2} < \beta < 2\pi$

(као у I групи): $\sin\alpha = -\frac{3}{5}$, $\cos\beta = \frac{7}{25}$

$\cos(\alpha+\beta) - \sin(\alpha-\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta - \sin\alpha\cos\beta + \cos\alpha\sin\beta =$

$= \cos\alpha(\sin\beta + \cos\beta) - \sin\alpha(\sin\beta + \cos\beta) = (\sin\beta + \cos\beta)(\cos\alpha - \sin\alpha) = \dots = \frac{17}{125}$

(I група)

③ Израчунати $\cos 3x$ у функцији од $\cos x$

$$\cos 3x = \cos(2x+x) = \cos 2x \cdot \cos x - \sin 2x \sin x =$$
$$= (\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \cdot \sin x = \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x$$
$$= \cos^3 x - 3\cdot \sin^2 x \cos x = \cos^3 x - 3(1-\cos^2 x)\cos x = 4\cos^3 x - 3\cos x$$

(II група) Израчунати $\sin 3x$ у функцији од $\sin x$

$$\sin 3x = \sin(2x+x) = \sin 2x \cdot \cos x + \cos 2x \cdot \sin x =$$
$$= 2\sin x \cos x \cdot \cos x + (\cos^2 x - \sin^2 x)\sin x = 2\sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x$$
$$= 3\sin x \cdot \cos^2 x - \sin^3 x = 3\sin x(1-\sin^2 x) - \sin^3 x = 3\sin x - 4\sin^3 x.$$

④ (II група) Израчунати тригонометријске ф-је половине угла, ако је $tg\,\alpha = \frac{4}{3}$, $0 < \alpha < \frac{\pi}{2}$.

$$\cos^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha + \cos^2\alpha} = \frac{1}{tg^2\alpha + 1} = \frac{1}{\frac{16}{9} + 1} = \frac{9}{25} \Rightarrow \cos\alpha = \frac{3}{5}$$

$$0 < \alpha < \frac{\pi}{2} \Rightarrow 0 < \frac{\alpha}{2} < \frac{\pi}{4}.$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\frac{3}{5}}{2} = \frac{\frac{2}{5}}{2} = \frac{1}{5} \Rightarrow \sin\frac{\alpha}{2} = \frac{\sqrt{5}}{5}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1+\frac{3}{5}}{2} = \frac{\frac{8}{5}}{2} = \frac{4}{5} \Rightarrow \cos\frac{\alpha}{2} = \frac{2\sqrt{5}}{5}$$

$$tg^2\frac{\alpha}{2} = \frac{1}{4} \Rightarrow tg\frac{\alpha}{2} = \frac{1}{2} \wedge ctg\frac{\alpha}{2} = 2.$$

(I група) Израчунати тригонометријске ф-је половине угла ако је $ctg\,\alpha = -\frac{7}{24}$, $\frac{\pi}{2} < \alpha < \pi$

$$\cos^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha + \cos^2\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha\left(1 + \frac{\cos^2\alpha}{\sin^2\alpha}\right)} = \frac{ctg^2\alpha}{1 + ctg^2\alpha} = \frac{\frac{49}{\cancel{576}}}{\frac{625}{\cancel{576}}} = \frac{49}{625}$$

$$\cos\alpha = -\frac{7}{25} \wedge \frac{\pi}{2} < \alpha < \pi \Rightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1+\frac{7}{25}}{2} = \frac{32}{50} = \frac{16}{25} \Rightarrow \sin\frac{\alpha}{2} = \frac{4}{5} \qquad tg\frac{\alpha}{2} = \frac{4}{3}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1-\frac{7}{25}}{2} = \frac{18}{50} = \frac{9}{25} \Rightarrow \cos\frac{\alpha}{2} = \frac{3}{5} \qquad ctg\frac{\alpha}{2} = \frac{3}{4}.$$

5. Доказати идентитет

(II група) $$\frac{1+\sin 2\alpha}{\cos 2\alpha} = \frac{tg\alpha + 1}{1 - tg\alpha}$$

$$L = \frac{1+2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha} =$$

$$= \frac{(\sin\alpha + \cos\alpha)^2}{(\cos\alpha - \sin\alpha)\cancel{(\cos\alpha + \sin\alpha)}} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} =$$

$$= \frac{\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} + 1\right)}{\cos\alpha\left(1 - \frac{\sin\alpha}{\cos\alpha}\right)} = \frac{tg\alpha + 1}{1 - tg\alpha} = D$$

(I група) $$\frac{\sin 2\alpha - \sin\alpha}{1-\cos\alpha + \cos 2\alpha} = tg\alpha$$

$$L = \frac{2\sin\alpha\cos\alpha - \sin\alpha}{1-\cos\alpha + \cos^2\alpha - \sin^2\alpha} = \frac{\sin\alpha(2\cos\alpha - 1)}{2\cos^2\alpha - \cos\alpha} =$$

$$= \frac{\sin\alpha(2\cos\alpha - 1)}{\cos\alpha(2\cos\alpha - 1)} = tg\alpha = D.$$