① (I) $\left(\left(\frac{5x^5}{2y^{-2}}\right)^{-2}\cdot\left(\frac{y^{-1}}{5x}\right)^{-3}\right):10x^{-2}y^{-3}=\frac{5^{-2}x^{-10}}{2^{-2}y^4}\cdot\frac{y^3}{5^{-3}x^{-3}}:\frac{10}{x^2y^3}$

$=\frac{2^2}{5^2y^4\cdot x^{10}}\cdot\frac{5^3y^3x^3}{1}:\frac{x^2y^3}{10}=\frac{2^2\cdot5^3x^5y^6}{2\cdot5^3\cdot x^{10}y^4}=\frac{2y^2}{x^5}$

(II) $\left(\left(\frac{2^{-1}}{3^{-1}a^{-3}b^3}\right)^4:\left(\frac{4a^{-2}}{3b^{-3}}\right)^{-3}\right)\cdot\frac{1}{12a^5b^{-2}}=\frac{2^{-1}}{3^{-4}a^{-12}b^{12}}:\frac{4^{-3}a^6}{3^{-3}b^9}\cdot\frac{b^2}{12a^5}=$

$=\frac{3^4a^{12}}{2^4b^{12}}:\frac{3^3a^6}{4^3b^9}\cdot\frac{b^2}{12a^5}=\frac{3^4\cdot2^6\cdot a^{12}\cdot b^{11}}{2^6\cdot3^4\cdot a^{11}\cdot b^{12}}=\frac{a}{b}$

② (I) $\frac{a^{-2}+b^{-2}}{a^{-1}+b^{-1}}\cdot\left(\frac{a^2+b^2}{ab}\right)^{-1}=\frac{\frac{1}{a^2}+\frac{1}{b^2}}{\frac{1}{a}+\frac{1}{b}}\cdot\frac{ab}{a^2+b^2}=\frac{\frac{a^2+b^2}{a^2b^2}}{\frac{a+b}{ab}}\cdot\frac{ab}{a^2+b^2}=$

$=\frac{a^2+b^2}{ab(a+b)}\cdot\frac{ab}{a^2+b^2}=(a+b)^{-1}$

(II) $\frac{a^{-4}-9b^{-2}}{a^{-2}-3b^{-1}}:\frac{(b+3a^2)^{-1}}{a^{-2}b^{-1}}=\frac{\frac{1}{a^4}-\frac{9}{b^2}}{\frac{1}{a^2}-\frac{3}{b}}\cdot\frac{a^2b}{b+3a^2}=$

$=\frac{\frac{b^2-9a^4}{a^4b^2}}{\frac{b-3a^2}{a^2b}}\cdot\frac{a^2b}{b+3a^2}=\frac{(b-3a^2)(b+3a^2)}{a^2b(b-3a^2)}\cdot\frac{a^2b}{b+3a^2}=1$

③ (II) $\left(\sqrt[3]{x^2\cdot\sqrt{x}}\left(\sqrt[3]{x^2}\right)^4\right):\sqrt{x^{-7}}=\left(\sqrt[6]{x^5}\cdot\sqrt[3]{x^8}\right):\sqrt{x^{-7}}=\sqrt[6]{\frac{x^5\cdot x^{16}}{x^{-21}}}=\sqrt[6]{x^{42}}=x^7$

(I) $(x\sqrt{x})^3\cdot\sqrt{x\sqrt[3]{x}}:x^4\sqrt[6]{x^5}=x^3\sqrt{x^3}\cdot\sqrt[6]{x^4}:x^4\sqrt[6]{x^5}=\frac{x^3}{x^4}\cdot\sqrt[6]{\frac{x^9\cdot x^4}{x^5}}=\frac{\sqrt[6]{x^8}}{x}=\sqrt[3]{x}$

④ (I) $\sqrt[6]{\frac{1}{a^2}-\frac{1}{a^3}+\frac{1}{a^4}}:\sqrt{\frac{a^2-a+1}{a^2}}=\sqrt[6]{\frac{(a^2-a+1)}{a^4}}\cdot\frac{a^{6\cdot2}}{(a^2-a+1)^{\frac{3}{2}}}=\sqrt[3]{\frac{a}{a^2-a+1}}$

(II) $\sqrt[4]{\frac{x}{x+2}+\frac{1}{(x+2)^2}}:\sqrt{\frac{2x+4}{x+2}}=\sqrt[4]{\frac{x^2+2x+1}{(x+2)^2}}\cdot\frac{1}{2^2}=\sqrt[4]{\left(\frac{x+1}{2(x+2)}\right)^2}=\sqrt{\frac{x+1}{2(x+2)}}$