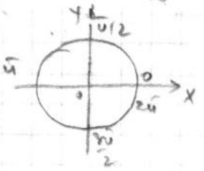
Контролна вежба – Гимназија 2 – 2018/2019

1) Упростити израз:

(I) $$\frac{\cos 620^\circ \cdot ctg\, 225^\circ \cdot \sin(-700)}{\sin 910^\circ \cdot tg(-855^\circ)\cdot \cos 1550^\circ} = \frac{-\sin 10 \cdot 1 \cdot \sin 20^\circ}{-\sin 10\cdot 1\cdot(-\sin 20)} = -1$$



$\cos 620^\circ = \cos(2\cdot 360 - 100^\circ) = \cos 100^\circ = \cos(90^\circ + 10) = -\sin 10^\circ$

$ctg\, 225^\circ = ctg(180^\circ + 45^\circ) = ctg\, 45^\circ = 1$

$\sin(-700^\circ) = -\sin(2\cdot 360 - 20^\circ) = -\sin(-20^\circ) = \sin 20^\circ$

$\sin 910^\circ = \sin(3\cdot 360 - 170^\circ) = -\sin(180^\circ - 10^\circ) = -\sin 10^\circ$

$tg(-855^\circ) = -tg(2\cdot 360^\circ + 135^\circ) = -tg(180^\circ - 45^\circ) = tg 45^\circ = 1$

$\cos 1550^\circ = \cos(4\cdot 360^\circ + 110^\circ) = \cos(90^\circ + 20^\circ) = -\sin 20^\circ$

(II) $$\frac{\sin 255^\circ \cdot tg\, 480^\circ \cdot \sin(-1490^\circ)}{\cos 615^\circ \cdot ctg\, 1230^\circ \cdot \cos 500^\circ} = \frac{\cancel{-\sin 15^\circ}\cdot(\cancel{-tg 60^\circ})\cdot(\cancel{-\sin 50^\circ})}{\cancel{-\sin 15^\circ}\cdot(\cancel{-ctg 30^\circ})\cdot(\cancel{-\sin 50^\circ})} = 1$$

$\sin 255^\circ = \sin(270^\circ - 15^\circ) = -\sin 15^\circ$

$tg\, 480^\circ = tg(360^\circ + 120) = tg\, 120^\circ = tg(180^\circ - 60^\circ) = -tg\, 60^\circ$

$\sin(-1490^\circ) = -\sin(4\cdot 360^\circ + 50^\circ) = -\sin 50^\circ$

$\cos 615^\circ = \cos(2\cdot 360^\circ - 105^\circ) = \cos(90^\circ + 15^\circ) = -\sin 15^\circ$

$ctg\, 1230^\circ = ctg(3\cdot 360^\circ + 150^\circ) = ctg(180^\circ - 30^\circ) = -ctg\, 30^\circ$

$\cos 500^\circ = \cos(360^\circ + 140^\circ) = \cos(90^\circ + 50^\circ) = -\sin 50^\circ$

2) Доказати:

(I) $$\frac{\sin(\pi - \alpha)}{\cos(\alpha - \frac{\pi}{2}) + \cos(2\pi - \alpha)} - \frac{\sin(\frac{5\pi}{2} + \alpha)}{\sin(\frac{\pi}{2} - \alpha) - \cos(\frac{\pi}{2} - \alpha)} = \frac{tg^2\alpha + 1}{tg^2\alpha - 1}$$

$$L = \frac{\sin\alpha}{\sin\alpha + \cos\alpha} - \frac{\cos\alpha}{\cos\alpha - \sin\alpha} = \frac{\sin\alpha(\sin\alpha - \cos\alpha) + \cos\alpha(\sin\alpha + \cos\alpha)}{\sin^2\alpha - \cos^2\alpha}$$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha - \cos^2\alpha} = \frac{tg^2\alpha + 1}{tg^2\alpha - 1} \equiv D$$

(II) $$\frac{\cos^2(\alpha - \frac{\pi}{2}) + ctg^2(\frac{\pi}{2} + \alpha) + 1}{\sin^2(\alpha - \frac{3\pi}{2}) + tg^2(\frac{3\pi}{2} + \alpha) + 1} = tg^2\alpha$$

$$L = \frac{\sin^2\alpha + tg^2\alpha + 1}{\cos^2\alpha + ctg^2\alpha + 1} = \frac{(\cancel{\sin^2\alpha\cdot\cos^2\alpha + \sin^2\alpha + \cos^2\alpha})\cdot \sin^2\alpha}{(\cancel{\sin^2\alpha\cdot\cos^2\alpha + \cos^2\alpha + \sin^2\alpha})\cdot\cos^2\alpha} = tg^2\alpha \equiv D$$

③ Одредити:

(I) $\sin 15$ ако је $\cos 15^\circ = \frac{1}{2}\sqrt{2+\sqrt{3}}$

$$\sin^2 15^\circ + \cos^2 15^\circ = 1$$

$$\sin^2 15^\circ = 1 - \frac{2+\sqrt{3}}{4} = \frac{2-\sqrt{3}}{4}$$

$$\sin 15^\circ = \frac{\sqrt{2-\sqrt{3}}}{2}$$

(II) $\cos 22^\circ 30'$ ако је $\sin 22^\circ 30' = \frac{\sqrt{2-\sqrt{2}}}{2}$

$$\sin^2 22^\circ 30' + \cos^2 22^\circ 30' = 1$$

$$\cos^2 22^\circ 30' = 1 - \frac{2-\sqrt{2}}{4} = \frac{2+\sqrt{2}}{4}$$

$$\cos 22^\circ 30' = \frac{\sqrt{2+\sqrt{2}}}{2}$$

④ (I) Доказати да вредност израза не зависи од a, b, α:

$$\frac{a^2\,tg(\pi+\alpha) + b^2\,ctg\left(\frac{3\pi}{2}+\alpha\right)}{a\cdot tg\left(\frac{\pi}{2}-\alpha\right) + b\cdot tg\left(\frac{3\pi}{2}+\alpha\right)} - (a+b)\cdot tg^2(2\pi-\alpha) =$$

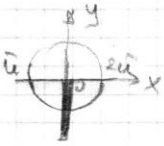
$$= \frac{a^2 tg\alpha - b^2\cdot tg\alpha}{a\cdot ctg\alpha - b\cdot ctg\alpha} - (a+b)\,tg^2\alpha = \frac{tg\alpha\,(a^2-b^2)}{ctg\alpha\,(a-b)} - (a+b)\,tg^2\alpha =$$

$$= \frac{tg^2\alpha\,\cancel{(a-b)}(a+b)}{\cancel{a-b}} - (a+b)\,tg^2\alpha = 0$$

(II) Ако је $\alpha-\beta = k\pi$, $k\in Z$, онда је:

$\sin(\alpha+\gamma)\cdot\sin(\alpha+\delta) = \sin(\beta+\gamma)\cdot\sin(\beta+\delta)$. Доказати.

Ако представимо $\alpha = k\pi+\beta$, биће:



$$\sin(\alpha+\gamma) = \sin(k\pi+\beta+\gamma) = -\sin(\beta+\gamma)$$

$$\sin(\alpha+\delta) = \sin(k\pi+\beta+\delta) = -\sin(\beta+\delta)$$

Сада је: $L \equiv -\sin(\beta+\gamma)\cdot(-\sin(\beta+\delta)) = \sin(\beta+\gamma)\sin(\beta+\delta) = D$