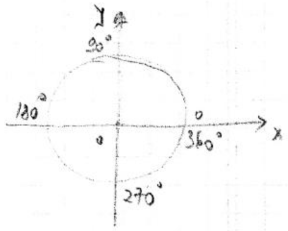
1) Упростити израз:

$$\frac{\sin(-750^\circ)\cdot\cos390^\circ\cdot tg(-1140^\circ)}{ctg(-495^\circ)\cdot\sin1860^\circ\cdot\cos540^\circ} \qquad \text{I}$$



$$\frac{\cos690^\circ\cdot\sin(-480^\circ)\cdot ctg\,1020^\circ}{tg\,405\cdot\sin1485^\circ\cdot\cos(-900^\circ)} \qquad \text{II}$$

(I)

$\sin(-750^\circ) = -\sin(30^\circ+720^\circ) = -\sin30 = -\frac{1}{2}$

$\cos390^\circ = \cos(360^\circ+30^\circ) = \cos30 = \frac{\sqrt3}{2}$

$tg(-1140^\circ) = -tg(3\cdot360^\circ+60^\circ) = -tg60^\circ = -\frac{\sqrt3}{3}$

$ctg(-495^\circ) = -ctg(360^\circ+135^\circ) = -ctg(180^\circ-45^\circ) = -(-ctg45^\circ) = 1$

$\sin1860^\circ = \sin(5\cdot360^\circ+60^\circ) = \sin60 = \frac{\sqrt3}{2}$

$\cos540^\circ = \cos(360^\circ+180^\circ) = \cos180^\circ = -1$

$$\frac{\sin(-750^\circ)\cdot\cos390^\circ\cdot tg(-1140^\circ)}{ctg405^\circ\cdot\sin1860^\circ\cdot\cos540^\circ} = \frac{-\frac{1}{2}\cdot\frac{\sqrt3}{\cancel{2}}\cdot\left(-\frac{\sqrt3}{3}\right)}{1\cdot\frac{\sqrt3}{\cancel{2}}\cdot(-1)} = -\frac{\sqrt3}{6}$$

(II)

$\cos690^\circ = \cos(2\cdot360^\circ-30^\circ) = \cos30 = \frac{\sqrt3}{2}$

$\sin(-480^\circ) = -\sin(360^\circ+120) = -\sin(180^\circ-60^\circ) = -\sin60^\circ = -\frac{\sqrt3}{2}$

$ctg1020^\circ = ctg(3\cdot360^\circ-60) = ctg(-60^\circ) = -ctg60^\circ = -\frac{\sqrt3}{3}$

$tg\,405^\circ = tg(360^\circ+45^\circ) = tg\,45^\circ = 1$

$\sin1485^\circ = \sin(4\cdot360^\circ+45^\circ) = \frac{\sqrt2}{2}$

$\cos(-900^\circ) = \cos(720^\circ+180^\circ) = \cos180^\circ = -1$

$$\frac{\cos690^\circ\cdot\sin(-480^\circ)\cdot ctg1020^\circ}{tg405^\circ\cdot\sin1485^\circ\cdot\cos(-900^\circ)} = \frac{\frac{\cancel{\sqrt3}}{2}\cdot\left(-\frac{\cancel{\sqrt3}}{\cancel{2}}\right)\cdot\left(-\frac{\sqrt3}{\cancel{3}}\right)}{1\cdot\frac{\sqrt2}{\cancel{2}}\cdot(-1)} = -\frac{\sqrt3}{2\sqrt2}\cdot\frac{\sqrt2}{\sqrt2} = -\frac{\sqrt6}{4}$$

② Упростити израз:

(I) $$A=\frac{ctg^2(\alpha-\frac{\pi}{2})\cdot\cos^2(\alpha-\frac{\pi}{2})}{ctg^2(\alpha-\frac{\pi}{2})-\cos^2(\alpha+\frac{5\pi}{2})}.$$

(II) $$B=\frac{tg(\frac{3}{2}\pi+\alpha)+tg^3(\frac{\pi}{2}-\alpha)}{ctg^3(\frac{5\pi}{2}-\alpha)+ctg(\frac{7\pi}{2}+\alpha)}$$

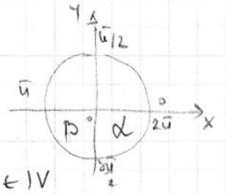
(I) $$A=\frac{tg^2\alpha\cdot\sin^2\alpha}{tg^2\alpha-\sin^2\alpha}=\frac{\frac{\sin^4\alpha}{\cos^2\alpha}}{\frac{\sin^2\alpha}{\cos^2\alpha}-\sin^2\alpha}=\frac{\frac{\sin^4\alpha}{\cos^2\alpha}}{\frac{\sin^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha}}=\frac{\sin^4\alpha}{\sin^4\alpha}=1$$

(II) $$B=\frac{-ctg\alpha+ctg^3\alpha}{tg^3\alpha-tg\alpha}=\frac{\frac{1}{tg^3\alpha}-\frac{1}{tg\alpha}}{tg^3\alpha-tg\alpha}=\frac{\frac{1-tg^2\alpha}{tg^3\alpha}}{tg\alpha(tg^2\alpha-1)}=-\frac{\cancel{tg^2\alpha-1}}{tg^4\alpha\cdot(\cancel{tg^2\alpha-1})}=$$
$$=-ctg^4\alpha.$$

③ Израчунати остале тригонометријске ф-је ако је

(I) $\cos\alpha=\frac{5}{13}$ и $\frac{3\pi}{2}<\alpha<2\pi$

(II) $\sin\beta=-\frac{1}{3}$ и $\pi<\beta<\frac{3\pi}{2}$



(I) $\sin^2\alpha+\cos^2\alpha=1$

$\sin^2\alpha=1-\frac{25}{169}$

$\sin\alpha=\pm\frac{12}{13}\ \wedge\ \alpha\in IV$

$\Downarrow$

$\boxed{\sin\alpha=-\frac{12}{13}}$

$tg\alpha=-\frac{12}{5}$, $ctg\alpha=-\frac{5}{12}$

(II) $\sin^2\beta+\cos^2\beta=1$

$\cos^2\beta=1-\frac{1}{9}$

$\cos\beta=\pm\frac{2\sqrt{2}}{3}\ \wedge\ \beta\in III$

$\Downarrow$

$\boxed{\cos\beta=-\frac{2\sqrt{2}}{3}}$

$tg\beta=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$, $ctg\beta=2\sqrt{2}$

④ Доказати: $$\frac{\cos^2\alpha+2\sin^2(\alpha-\pi)}{\cos^3(\alpha-4\pi)}+\frac{\cos^2\alpha+4\sin\alpha+\sin^2(\alpha+\pi)}{\cos\alpha\cdot(4\sin\alpha+1)}=\frac{2}{\cos^3\alpha}$$

$$L\equiv\frac{\cos^2\alpha+2\sin^2\alpha}{\cos^3\alpha}+\frac{\cos^2\alpha+4\sin\alpha+\sin^2\alpha}{\cos\alpha(4\sin\alpha+1)}=\frac{1+\sin^2\alpha}{\cos^3\alpha}+\frac{\cancel{1+4\sin\alpha}}{\cos\alpha(\cancel{4\sin\alpha+1})}$$

$$=\frac{1+\sin^2\alpha+\cos^2\alpha}{\cos^3\alpha}=\frac{2}{\cos^3\alpha}$$