

① НАПИСАТИ КВАДРАТНУ ЈЕДНАЧИНУ ЧИЈА СУ РЕШЕЊА

(I) $x_1 = \frac{3-2}{3-i\sqrt{5}}$ $x_2 = \frac{2}{3+i\sqrt{5}}$

$x_1 = \frac{2(3+i\sqrt{5})}{14} = \frac{3+i\sqrt{5}}{7}$

$x_2 = \frac{2(3-i\sqrt{5})}{14} = \frac{3-i\sqrt{5}}{7}$

$x_1 + x_2 = \frac{6}{7} \Rightarrow b = -\frac{6}{7}a$

$x_1 \cdot x_2 = \frac{14}{49} \Rightarrow c = \frac{2}{7}a$

$7x^2 - 6x + 2 = 0$

(II) $x_1 = \frac{3}{\sqrt{6}+2}$ $x_2 = \frac{3}{\sqrt{6}-2}$

$x_1 = \frac{3(\sqrt{6}-2)}{2}$ $x_2 = \frac{3(\sqrt{6}+2)}{2}$

$x_1 + x_2 = \frac{3(\sqrt{6}-2+\sqrt{6}+2)}{2} = 3\sqrt{6}$

$x_1 \cdot x_2 = \frac{9}{4} (6-4) = \frac{9}{4} \cdot 2 = \frac{9}{2}$

$b = -3\sqrt{6}a$, $c = \frac{9}{2}a$

$2x^2 - 6\sqrt{6}x + 9 = 0$

② ДОКАЗАТИ ДА СУ ДАТЕ ФАКТОРИ

(I) РЕГУПРОЧТИ

$u = \frac{m^2x^2 + 3mx + 2}{m^2x^2 + 5mx + 6} = \frac{(mx+1)(mx+2)}{(mx+3)(mx+2)}$

$v = \frac{m^2x^2 - 2mx - 15}{m^2x^2 - 4mx - 5} = \frac{(mx-5)(mx+3)}{(mx-5)(mx+1)}$

$m^2x^2 + 3mx + 2 = m^2(x + \frac{1}{m})(x + \frac{2}{m})$

$x_{1,2} = \frac{-3m \pm m}{2m^2} = \left\{ \begin{array}{l} -\frac{1}{m} \\ -\frac{2}{m} \end{array} \right.$

$m^2x^2 + 5mx + 6 = m^2(x + \frac{3}{m})(x + \frac{2}{m})$

$x_{1,2} = \frac{-5m \pm m}{2m^2} = \left\{ \begin{array}{l} -\frac{3}{m} \\ -\frac{2}{m} \end{array} \right.$

$m^2x^2 - 2mx - 15 = m^2(x - \frac{5}{m})(x + \frac{3}{m})$

$x_{1,2} = \frac{2m \pm 8m}{2m^2} = \left\{ \begin{array}{l} \frac{5}{m} \\ -\frac{3}{m} \end{array} \right.$

$m^2x^2 - 4mx - 5 = m^2(x - \frac{5}{m})(x + \frac{1}{m})$

$x_{1,2} = \frac{4m \pm 6m}{2m^2} = \left\{ \begin{array}{l} \frac{5}{m} \\ -\frac{1}{m} \end{array} \right.$

$u \cdot v = 1$

(II) МЕТОД ДЕЛИКА :

$P = \frac{10x^2 - 3mx - m^2}{6x^2 - 11mx + 4m^2} = \frac{(2x-m)(5x+m)}{(3x-4m)(2x-m)}$

$Q = \frac{15x^2 - 17mx - 4m^2}{9x^2 - 24mx + 16m^2} = \frac{(3x-4m)(5x+m)}{(3x-4m)^2}$

$10x^2 - 3mx - m^2 = 10(x - \frac{m}{2})(x + \frac{m}{5})$

$x_{1,2} = \frac{3m \pm 7m}{20} = \left\{ \begin{array}{l} \frac{m}{2} \\ -\frac{m}{5} \end{array} \right.$

$6x^2 - 11mx + 4m^2 = 6(x - \frac{4m}{3})(x - \frac{m}{2})$

$x_{1,2} = \frac{11m \pm 5m}{12} = \left\{ \begin{array}{l} \frac{4m}{3} \\ \frac{m}{2} \end{array} \right.$

$15x^2 - 17mx - 4m^2 = 15(x - \frac{4}{3}m)(x + \frac{m}{5})$

$x_{1,2} = \frac{17m \pm 23m}{30} = \left\{ \begin{array}{l} \frac{4}{3}m \\ \frac{m}{5} \end{array} \right.$

$9x^2 - 24mx + 16m^2 = (3x - 4m)^2$

$P = Q$

③ Решить уравнение.

$$(I) \frac{x^2}{2(2x+1)} = \frac{2(2x-1)}{x^2}$$

$$(x^2)^2 = 4(4x^2-1), x \neq 0, x \neq -\frac{1}{2}$$

$$t^2 - 16t + 4 = 0, x^2 = t$$

$$t_{1,2} = \frac{16 \pm 4\sqrt{15}}{2} = 8 \pm 2\sqrt{15}$$

$$t_{1,2} = 5 \pm 3 \pm 2\sqrt{5} \cdot \sqrt{3} = (\sqrt{5} \pm \sqrt{3})^2$$

$$x^2 = (\sqrt{5} \pm \sqrt{3})^2 \Rightarrow x_{1,2} = \pm(\sqrt{5} \pm \sqrt{3})$$

$$(II) \frac{x^2(3x+4)}{2x+1} = \frac{3(2x-1)}{3x-4}$$

$$x^2(9x^2-16) = 3(4x^2-1), x \neq -\frac{1}{2}, x \neq \frac{4}{3}$$

$$9x^4 - 16x^2 = 12x^2 - 3$$

$$9x^4 - 28x^2 + 3 = 0, x^2 = t$$

$$t_{1,2} = \frac{28 \pm \sqrt{28^2 - 12 \cdot 9}}{18} = \frac{28 \pm 26}{18} = \begin{cases} 3 \\ \frac{1}{9} \end{cases}$$

$$x_{1,2} = \pm\sqrt{3}, x_{3,4} = \pm\frac{1}{3}$$

④ Решить систему

$$(I) \begin{cases} x^2 + y = 13 \\ x^2 + y^2 = 25 \end{cases}$$

$$x^2 + y^2 = 25$$

$$x^2 = 13 - y$$

$$y^2 - y - 12 = 0$$

$$y_{1,2} = \frac{1 \pm 7}{2} = \begin{cases} 4 \\ -3 \end{cases}$$

$$x^2 = 13 - y$$

$$y_1 = 4: x_{1,2} = \pm 3$$

$$y_2 = -3: x_{3,4} = \pm 4$$

$$(3, 4), (-3, 4), (4, -3), (-4, -3)$$

$$(II) \begin{cases} x^2 - y^2 = 5 \\ x^2 + y = 11 \end{cases}$$

$$x^2 + y = 11$$

$$x^2 = 5 + y^2$$

$$5 + y^2 + y - 11 = 0$$

$$x^2 = 5 + y^2$$

$$y^2 + y - 6 = 0$$

$$y_{1,2} = \frac{-1 \pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

$$y_1 = 2: x_{1,2} = \pm 3$$

$$y_2 = -3: x_{3,4} = \pm\sqrt{14}$$

$$(3, 2), (-3, 2), (\sqrt{14}, -3), (-\sqrt{14}, -3)$$