

Eksponecijalne jednačine

1. Rešiti jednačine

$$(a) 2^x = 64 \quad (b) 3^{x-3} = 27 \quad (c) 2^{x-1} = \frac{1}{4} \quad (d) 3^{x-1} = 1 \quad (e) 3^{2x-1} = \frac{1}{9}$$
$$(f) \left(\frac{1}{4}\right)^x = \frac{1}{16} \quad (g) \left(\frac{1}{3}\right)^{2x-3} = 27 \quad (h) 2^{x^2-x} = 2^6 \quad (i) 5^{x^2+2x} = 125 \quad (j) 2^{-x^2+9x-8} = 8^{-x^2-1}$$
$$(k) 7^{x-1} = \sqrt[4]{7} \quad (l) 2^{3x-1} = \sqrt[4]{32} \quad (m) \left(\frac{3}{5}\right)^{2x-5} = \left(\frac{5}{3}\right)^{3x} \quad (n) \left(\frac{9}{4}\right)^{2x-1} = \left(\frac{2}{3}\right)^{x+1} \quad (o) 3^{x^2} = \frac{1}{3^{2-3x}}$$

rešenje:

$$(a) x = 6 \quad (b) x = 6 \quad (c) x = -1 \quad (d) x = 1 \quad x = -\frac{1}{2}$$
$$(f) x = 2 \quad (g) x = 0 \quad (h) x = -2, x = 3 \quad (i) x = -3, x = 1 \quad (j) x = -5, x = \frac{1}{2}$$
$$(k) x = \frac{5}{4} \quad (l) x = \frac{3}{4} \quad (m) x = 1 \quad (n) x = \frac{1}{5} \quad (o) x = 1, x = 2$$

2. Rešiti jednačine

$$(a) 2^x \cdot 3^x = 36 \quad (b) 2^{x+1} \cdot 3^x = 72 \quad (c) 2^x \cdot 5^x = \frac{1}{10} \quad (d) 2^x \cdot 5^x = 1000$$

rešenje:

$$(a) x = 2 \quad (b) x = 2 \quad (c) x = -1 \quad (d) x = 3$$

3. Rešiti jednačine

$$(a) 2^{x-1} \cdot 4^x = 32 \quad (b) 3^{2x-1} \cdot 27^{\frac{x}{2}} = 1 \quad (c) (\sqrt{2})^{4x-1} \cdot 4^{x+\frac{1}{2}} = \frac{1}{2}$$
$$(d) \sqrt{3^x} \cdot \left(\frac{1}{3}\right)^{x+\frac{1}{2}} = 3 \quad (e) 16 \cdot 2^{5x+2} = 2^{x^2} \quad (f) (9^{x-1})^{x-1} = 9^{x-4} \cdot 3^{2x+4}$$

rešenje:

$$(a) x = 2 \quad (b) x = \frac{2}{7} \quad (c) x = -\frac{3}{8}$$
$$(d) x = -3 \quad (e) x = -1, x = 6 \quad (f) x = 1, x = 3$$

4. Rešiti jednačine

$$(a) 2^x \cdot 5^{x+2} = 2500 \quad (b) 2^{x-1} \cdot 3^{x+1} = 54 \quad (c) 2^{x+\frac{1}{2}} \cdot 5^{x+1} = 500\sqrt{2}$$

rešenje:

$$(a) x = 2 \quad (b) x = 2 \quad (c) x = 2$$

5. Rešiti jednačine

$$(a) 3^{x+2} + 3^{x+1} = 324 \quad (b) 3^{x-1} + 5 \cdot 3^{x+1} = 138 \quad (c) \left(\frac{1}{2}\right)^{x-3} + 6 \cdot \left(\frac{1}{2}\right)^{x+1} = \frac{11}{8}$$

rešenje:

$$(a) x = 3 \quad (b) x = 2 \quad (c) x = 3$$

6. Rešiti jednačine

$$(a) 16^x - 3 \cdot 4^x + 2 = 0 \quad (b) 9^x - 4 \cdot 3^x = -3 \quad (c) 2^{2x+1} - 33 \cdot 2^{x-1} + 4 = 0$$
$$(d) 5^{2x-3} - 2 \cdot 5^{x-2} = 3 \quad (e) 9^{x^2-1} - 36 \cdot 3^{x^2-3} + 3 = 0 \quad (f) 2^{x+\sqrt{x^2-4}} - 5 \cdot (\sqrt{2})^{x-2+\sqrt{x^2-4}} - 6 = 0$$

rešenje:

$$(a) x = 0, x = \frac{1}{2} \quad (b) x = 0, x = 1 \quad (c) x = -2, x = 3$$

$$(d) x = 2 \quad (e) x = \pm 1, x = \pm \sqrt{2} \quad (f) x = \frac{5}{2}$$

7. Rešiti jednačine

$$(a) 30 \cdot 25^x - 5 \cdot 150^x + 6^x = 6 \quad (b) 2^{2x+1} - 5 \cdot 6^x + 3^{2x+1} = 0 \quad (c) 7 \cdot 9^x - 10 \cdot 21^x + 3 \cdot 49^x = 0$$

rešenje:

$$(a) x = -\frac{1}{2}, x = 1 \quad (b) x = 0, x = -1 \quad (c) x = 0, x = 1$$

8. Rešiti jednačine

$$(a) 7 \cdot 3^{x+1} - 5^{x+2} = 3^{x+4} - 5^{x+3} \quad (b) 7^{x-1} + 7^{x-2} = 2^x + 2^{x+1} + 2^{x+2} \quad (c) 6^x + 6^{x+1} = 2^x + 2^{x+1} + 2^{x+2}$$

$$(d) 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1} \quad (e) 3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^{x+1} - \frac{1}{2} \cdot 9^{x+1} \quad \square$$

rešenje:

$$(a) x = -1 \quad (b) x = 3 \quad (c) x = 0$$

$$(d) x = \frac{3}{2} \quad (e) \frac{1}{2} \quad \square$$

9. Rešiti jednačine

$$(a) \left(\sqrt{2-\sqrt{3}}\right)^x + \left(\sqrt{2+\sqrt{3}}\right)^x = 4, \quad (b) \left(\sqrt{7+\sqrt{48}}\right)^x + \left(\sqrt{7-\sqrt{48}}\right)^x = 14$$

rešenje: (a) $x = -2, x = 2$, (b) $x = 2, x = -2$

Eksponecijalne nejednačine

10. Odrediti skup rešenja nejednačina

$$(a) 2^{3-6x} > 1 \quad (b) 16^x > \frac{1}{8} \quad (c) (0.1)^{4x^2-2x-2} \leq (0.1)^{2x-3} \quad (d) \left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{25}{4}$$

rešenje:

$$(a) x \in (-\infty, \frac{1}{2}) \quad (b) x \in (-\frac{3}{4}, \infty) \quad (c) x \in \mathbb{R} \quad (d) x \in (-\infty, -2) \cup (-\frac{2}{5}, \infty)$$

Sistemi eksponecijalnih jednačina

11. Rešiti sistem jednačina

$$(a) \begin{cases} 3^x - 3 \cdot 2^y = -11 \\ 4 \cdot 3^x + 2^y = 8 \end{cases} \quad (b) \begin{cases} 2 \cdot 3^{x+1} - 2^{y+1} = 17 \\ 3^x + 2^{y+1} = 4 \end{cases} \quad (c) \begin{cases} 3^{2x} - 2^y = 77 \\ 3^x - 2^{\frac{y}{2}} = 7 \end{cases}$$

rešenje:

$$(a) x = 0, y = 2 \quad (b) x = 1, y = -1 \quad (c) x = 2, y = 2$$