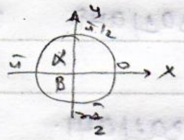
1° (I) Ако је $\sin\alpha = \frac{4}{5}$ и $\cos\beta = -\frac{24}{25}$ и $\frac{\pi}{2} < \alpha < \pi$, $\pi < \beta < \frac{3\pi}{2}$ израчунати $\cos(\pi - \alpha + \beta)$.

(II) Ако је $\cos\alpha = \frac{12}{13}$ и $\sin\beta = -\frac{3}{5}$ и $\frac{\pi}{2} < \alpha < \pi$, $\pi < \beta < \frac{3\pi}{2}$ израчунати $\sin(\pi + \alpha - \beta)$.

решење:



(I) $\cos^2\alpha = 1 - \frac{16}{25} = \frac{9}{25} \Rightarrow \cos\alpha = -\frac{3}{5}$;

$\sin\beta = -\frac{7}{25}$

$\cos(\pi - \alpha + \beta) = \cos(\pi - (\alpha - \beta)) = -\cos(\alpha - \beta) = -\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\left(-\frac{3}{5}\right)\cdot\left(-\frac{24}{25}\right) - \frac{4}{5}\cdot\left(-\frac{7}{25}\right) = -\frac{72}{125} + \frac{28}{125} = -\frac{44}{125}$

(II) Аналогно као за прву групу, али пазити на квадранте

$\sin\alpha = \frac{5}{13}$; $\cos\beta = -\frac{4}{5}$

$\sin(\pi + \alpha - \beta) = -\sin(\alpha - \beta) = -\sin\alpha\cos\beta + \cos\alpha\sin\beta =$

$= -\frac{5}{13}\cdot\left(-\frac{4}{5}\right) + \frac{12}{13}\cdot\left(-\frac{3}{5}\right) = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$.

2° (I) Доказати да је $\frac{\cos\frac{x}{2} - \cos x - 1}{\sin\frac{x}{2} - \sin x} = \operatorname{ctg}\frac{x}{2}$

(II) Доказати да је $\frac{1 + \sin x - \cos x}{1 + \sin x + \cos x} = \operatorname{tg}\frac{x}{2}$

решење:

(I) $L = \frac{\cos\frac{x}{2} - (1 + \cos x)}{\sin\frac{x}{2} - \sin 2\cdot\frac{x}{2}} = \frac{\cos\frac{x}{2} - 2\cdot\frac{1+\cos x}{2}}{\sin\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{\cos\frac{x}{2} - 2\cos^2\frac{x}{2}}{\sin\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}} =$

$= \frac{\cos\frac{x}{2}\cancel{\left(1 - 2\cos\frac{x}{2}\right)}}{\sin\frac{x}{2}\cancel{\left(1 - 2\cos\frac{x}{2}\right)}} = \operatorname{ctg}\frac{x}{2} \equiv D.$

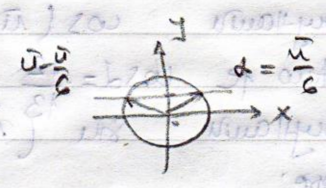
(II) $L = \frac{1 - \cos x + \sin x}{1 + \cos x + \sin x} = \frac{2\cdot\frac{1-\cos x}{2} + \sin 2\cdot\frac{x}{2}}{2\cdot\frac{1+\cos x}{2} + \sin 2\cdot\frac{x}{2}} = \frac{2\sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}} =$

$= \frac{\cancel{2}\sin\frac{x}{2}\cancel{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)}}{\cancel{2}\cos\frac{x}{2}\cancel{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}} = \operatorname{tg}\frac{x}{2} \equiv D.$

(3) Решити тригонометријску једначину

(I) $2\cdot\sin\left(3x-\frac{\pi}{3}\right)=1$

(II) $2\cdot\cos\left(2x+\frac{\pi}{3}\right)=\sqrt{2}$



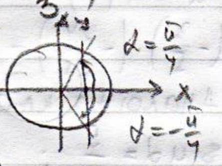
решење: (I) $\sin\left(3x-\frac{\pi}{3}\right)=\frac{1}{2}$

I квадрант: $3x-\frac{\pi}{3}=\frac{\pi}{6}+2k\pi,\ k\in Z$

$3x=\frac{\pi}{2}+2k\pi$ тј. $x_1=\frac{\pi}{6}+\frac{2k\pi}{3},\ k\in Z$

II квадрант: $3x-\frac{\pi}{3}=\pi-\frac{\pi}{6}+2l\pi,\ l\in Z$

$3x=\frac{7\pi}{6}+2l\pi$ тј. $x_2=\frac{7\pi}{18}+\frac{2l\pi}{3},\ l\in Z$



(II) $\cos\left(2x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$

I квадрант: $2x+\frac{\pi}{3}=\frac{\pi}{4}+2k\pi,\ k\in Z$

$2x=-\frac{\pi}{12}+2k\pi$ тј. $x_1=-\frac{\pi}{24}+k\pi,\ k\in Z$

IV квадрант: $2x+\frac{\pi}{3}=-\frac{\pi}{4}+2l\pi,\ l\in Z$

$2x=-\frac{7\pi}{12}+2l\pi$ тј. $x_2=-\frac{7\pi}{24}+l\pi,\ l\in Z.$

(4) Доказати да израз $\cos^2x-2\sin\alpha\cos x\cdot\sin(\alpha+x)+\sin^2(\alpha+x)$ не зависи од x.

Раставимо све: $\cos^2x-2\sin\alpha\cos x\,(\sin\alpha\cos x+\cos\alpha\sin x)+(\sin\alpha\cos x+\cos\alpha\sin x)^2=$

$=\cos^2x-2\sin^2\alpha\cos^2x-\cancel{2\sin\alpha\cos\alpha\sin x\cos x}+\sin^2\alpha\cos^2x+\cancel{2\sin\alpha\cos\alpha\sin x\cos x}+$

$+\cos^2\alpha\sin^2x=\cos^2x-\sin^2\alpha\cos^2x+\cos^2\alpha\sin^2x=$

$=\cos^2x(1-\sin^2\alpha)+\cos^2\alpha\sin^2x=\cos^2x\cdot\cos^2\alpha+\cos^2\alpha\sin^2x=$

$=\cos^2\alpha(\cos^2x+\sin^2x)=\cos^2\alpha$

По решењу се види да израз не зависи од x.