1. Одредити тригонометријске ф-је половине угла, ако:

(I) $\operatorname{tg}\alpha = \frac{12}{5}$, $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$

$\sin\alpha = \frac{12}{5}\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{144}{25}\cos^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{25}{169} \Rightarrow \cos\alpha = -\frac{5}{13}$

$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{\frac{18}{13}}{2} = \frac{9}{13}$, $\frac{\alpha}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$\boxed{\sin\frac{\alpha}{2} = \frac{3\sqrt{13}}{13}}$; $\cos^2\frac{\alpha}{2} = \frac{\frac{8}{13}}{2} = \frac{4}{13}$; $\boxed{\cos\frac{\alpha}{2} = -\frac{2\sqrt{13}}{13}}$

$\operatorname{tg}\frac{\alpha}{2} = -\frac{3}{2}$; $\operatorname{ctg}\frac{\alpha}{2} = -\frac{2}{3}$

(II) $\operatorname{ctg}\alpha = -\frac{7}{24}$, $\alpha \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\sin\alpha = -\frac{24}{7}\cos\alpha$ јер је $\operatorname{tg}\alpha = -\frac{24}{7}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{576}{49} + 1\right)\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{49}{625}$

$\cos\alpha = \pm\frac{7}{25}$, $\alpha \in IV \Rightarrow \boxed{\cos\alpha = \frac{7}{25}}$

$\sin^2\frac{\alpha}{2} = \frac{\frac{18}{25}}{2} = \frac{9}{25} \Rightarrow \sin\frac{\alpha}{2} = \pm\frac{3}{5}$

$\frac{\alpha}{2} \in \left(\frac{3\pi}{4}, \pi\right)$ П. $\sin\frac{\alpha}{2} = +\frac{3}{5}$

$\cos^2\frac{\alpha}{2} = \frac{1+\frac{7}{25}}{2} = \frac{16}{25}$ П. $\boxed{\cos\frac{\alpha}{2} = -\frac{4}{5}}$

$\operatorname{tg}\frac{\alpha}{2} = -\frac{3}{4}$, $\operatorname{ctg}\frac{\alpha}{2} = -\frac{4}{3}$

2. Израчунај $\sin 2\alpha$, $\cos 2\alpha$, $\operatorname{tg} 2\alpha$, ако је:

(I) $\cos\alpha = -\frac{5}{13}$, $\alpha \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$\sin^2\alpha = 1 - \frac{25}{169} = \frac{144}{169} \Rightarrow \sin\alpha = \frac{12}{13}$

$\sin 2\alpha = 2\cdot\frac{12}{13}\cdot\left(-\frac{5}{13}\right) = -\frac{120}{169}$

$\cos 2\alpha = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$ јер $2\alpha \in \left(\pi, \frac{3\pi}{2}\right)$

$\operatorname{tg} 2\alpha = \frac{120}{119}$.

(II) $\sin\alpha = 0{,}6$, $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\cos^2\alpha = 0{,}64 \Rightarrow \cos\alpha = \pm 0{,}8$

$\cos\alpha = 0{,}8$

$\sin 2\alpha = 2\cdot 0{,}6\cdot 0{,}8 = 0{,}96$

$\cos 2\alpha = 0{,}64 - 0{,}36 = 0{,}28$

$\operatorname{tg} 2\alpha = \frac{24}{7}$.

3. **(I)** Ако је $\operatorname{tg}\alpha = \frac{1}{7}$ и $\alpha + \beta = \frac{\pi}{4}$, нађи $\operatorname{tg}\beta$

$\operatorname{tg}\beta = \operatorname{tg}\left(\frac{\pi}{4} - \alpha\right) = \frac{\operatorname{tg}\frac{\pi}{4} - \operatorname{tg}\alpha}{1 + \operatorname{tg}\frac{\pi}{4}\cdot\operatorname{tg}\alpha} = \frac{1 - \frac{1}{7}}{1 + \frac{1}{7}} = \frac{6}{8} =$

$\boxed{\operatorname{tg}\beta = \frac{3}{4}}$

(II) Ако је $\alpha + \beta = 60^\circ$ и $\cos\alpha = \frac{11}{13}$, нађи $\cos\beta$.

$\cos\beta = \cos(60^\circ - \alpha) = \cos 60^\circ \cos\alpha + \sin 60^\circ \sin\alpha =$

$= \frac{1}{2}\cdot\frac{11}{13} + \frac{\sqrt{3}}{2}\cdot\frac{4\sqrt{3}}{13} = \frac{11}{26} + \frac{12}{26} = \frac{23}{26}$.

$\sin^2\alpha = 1 - \cos^2\alpha = \frac{48}{169} \Rightarrow \sin\alpha = \frac{4\sqrt{3}}{13}$

4. Упростити израз:

(I) $\frac{\sin 750^\circ \cdot \cos 390^\circ \cdot \operatorname{tg} 1140^\circ}{\operatorname{ctg} 405^\circ \cdot \sin 1860^\circ \cdot \cos 780^\circ} = \ldots = \sqrt{3}$

(II) $\frac{\cos\frac{17\pi}{6} \cdot \sin\frac{7\pi}{3} \cdot \operatorname{tg}\frac{17\pi}{4}}{\operatorname{ctg}\frac{10\pi}{3} \cdot \cos\frac{7\pi}{4} \cdot \sin\frac{8\pi}{3}} = \ldots = -\frac{3}{2}\sqrt{2}$

5. Доказати идентитет

$\cos^4\alpha + \sin^4\alpha = 1 - 0{,}5\sin^2 2\alpha$

$L \equiv (\cos^2\alpha)^2 + (\sin^2\alpha)^2 + 2\sin^2\alpha\cos^2\alpha - 2\sin^2\alpha\cos^2\alpha = (\cos^2\alpha + \sin^2\alpha)^2 - \frac{4}{2}\sin^2\alpha\cos^2\alpha$

$= 1 - \frac{1}{2}(2\sin\alpha\cos\alpha)^2 = 1 - 0{,}5\cdot\sin^2 2\alpha \equiv D$.