ИЗРАЧУНАТИ ИНТЕГРАЛЕ:

ПРВА ГРУПА

IV₂

1) $\int_{\pi/4}^{\pi/2} \mathrm{ctg}\, x \, dx$

2) $\int_{1/2}^{1} \arcsin x \, dx$

3) $\int_{0}^{\pi/4} \frac{x \sin x}{\cos^3 x} \, dx$

$\boxed{\mathrm{IV}_1}$

1) $\int_{\pi/6}^{\pi} \mathrm{tg}\, x \, dx$

2) $\int_{1}^{\ln 2} (2x+1)e^{x} \, dx$

3) $\int_{1/3}^{\sqrt{3}/3} \frac{dx}{\sqrt{4-9x^2}}$

ДРУГА ГРУПА

IV₂

1) $\int_{0}^{\pi/3} \sin^3 x \, dx$

2) $\int_{0}^{1} \mathrm{arctg}\, x \, dx$

3) $\int_{0}^{3} \sqrt{9-x^2} \, dx$

$\boxed{\mathrm{IV}_1}$

1) $\int_{0}^{\pi/6} \cos^3 x \, dx$

2) $\int_{1}^{e^2} x \ln x \, dx$

3) $\int_{1}^{\sqrt[6]{3}} \frac{x^2 \, dx}{1+x^6}$

Одредити површину фигуре омеђене линијама

4) $y=e^{x}, \; y=e^{-x}, \; x=2$

4) $y=\ln x, \; y=0, \; x=e.$

① Метода смене код одређеног интеграла

I група

$$\int_{\pi/6}^{\pi} tg\,x\,dx = \int_{\pi/6}^{\pi} \frac{\sin x}{\cos x}\,dx = \left(\begin{array}{l} \text{смена: } \cos x = t \\ -\sin x\,dx = dt \\ x=\pi: t=1;\ x=\frac{\pi}{6}: t=\frac{\sqrt3}{2} \end{array}\right) =$$

$$= -\int_{\sqrt3/2}^{1} \frac{dt}{t} = -\ln|t|\Big|_{\sqrt3/2}^{1} = \ln\frac{\sqrt3}{2}.$$

II група

$$\int_0^{\pi/6} \cos^3 x\,dx = \int_0^{\pi/6} \cos x\,(1-\sin^2 x)\,dx = \int_0^{\pi/6}\cos x\,dx - \int_0^{\pi/6}\cos x \sin^2 x\,dx$$

$$= \sin x\Big|_0^{\pi/6} - \int_0^{\pi/6}\cos x\sin^2 x\,dx = \left(\begin{array}{ll} \text{смене: } \sin x = t & x=0: t=0 \\ \cos x\,dx = dt & x=\frac{\pi}{6}: t=\frac12 \end{array}\right) =$$

$$= \frac12 - \int_0^{1/2} t^2\,dt = \frac12 - \frac{t^3}{3}\Big|_0^{1/2} = \frac12 - \frac13\cdot\frac18 = \frac12 - \frac1{24} = \frac{11}{24}.$$

② Метода парцијалне интеграције код одређеног интеграла.

I група

$$\int_1^{\ln 2}(2x+1)e^x\,dx = \left(\begin{array}{l} u=2x+1;\ du=2\,dx \\ dv=e^x dx;\ v=e^x \end{array}\right) =$$

$$= (2x+1)e^x\Big|_1^{\ln 2} - 2\int_1^{\ln 2} e^x\,dx = (2x+1)e^x\Big|_1^{\ln 2} - 2e^x\Big|_1^{\ln 2} =$$

$$= (2x-1)e^x\Big|_1^{\ln 2} = (2\ln 2-1)\cdot 2 - e = (\ln 4-1)2 - e = 2\ln 4 - 2 - e$$

II група

$$\int_1^{e^2} x\ln x\,dx = \left(\begin{array}{l} u=\ln x;\ du=\frac1x\,dx \\ dv=x\,dx;\ v=\frac{x^2}{2} \end{array}\right) =$$

$$= \frac{x^2}{2}\ln x\Big|_1^{e^2} - \frac12\int_1^{e^2} x\,dx = \frac{x^2}{2}\ln x\Big|_1^{e^2} - \frac12\,\frac{x^2}{2}\Big|_1^{e^2} = \frac{x^2}{2}\left(\ln x - \frac12\right)\Big|_1^{e^2} =$$

$$= \frac{e^4}{2}\cdot\frac32 - \frac12\left(-\frac12\right) = \frac{3e^4}{4} + \frac14 = \frac{3e^4+1}{4}$$

③ Комбинован пример

I група

$$\int_{1/3}^{\sqrt3/3}\frac{dx}{\sqrt{4-9x^2}} = \frac12\int_{1/3}^{\sqrt3/3}\frac{dx}{\sqrt{1-\left(\frac{3x}{2}\right)^2}} = \left(\begin{array}{l} \text{смена } \frac{3x}{2}=t \\ dx=\frac23\,dt \\ x=\frac13: t=\frac12;\ x=\frac{\sqrt3}{3}: t=\frac{\sqrt3}{2} \end{array}\right) =$$

$$= \frac12\cdot\frac23\int_{1/2}^{\sqrt3/2}\frac{dt}{\sqrt{1-t^2}} = \frac13\arcsin t\Big|_{1/2}^{\sqrt3/2} = \frac13\left(\frac\pi3 - \frac\pi6\right) = \frac13\cdot\frac\pi6 = \frac{\pi}{18}.$$

II група

$$\int_1^{\sqrt[6]{3}}\frac{x^2\,dx}{1+x^6} = \int_1^{\sqrt[6]{3}}\frac{x^2\,dx}{1+(x^3)^2} = \left(\begin{array}{ll} \text{смена: } x^3=t & x=1: t=1 \\ 3x^2dx=dt; & x=\sqrt[6]{3}: t=\sqrt3 \end{array}\right) =$$

$$= \frac13\int_1^{\sqrt3}\frac{dt}{1+t^2} = \frac13\,\text{arctg}\,t\Big|_1^{\sqrt3} = \frac13\left(\frac\pi3 - \frac\pi4\right) = \frac13\cdot\frac{\pi}{12} = \frac{\pi}{36}.$$

* Решења четвртог задатка су у решењима одељења IV_2 *