

Прва група

1. Упростити израз: $\frac{\cos(\pi+\alpha)\sin(\frac{3\pi}{2}+\alpha)}{\sin(\frac{\pi}{2}-\alpha)\cos(\alpha-3\pi)}$
2. Упростити израз: $\frac{\sin(-\frac{3\pi}{2})\operatorname{tg}\frac{7\pi}{3}\cos 770^\circ}{\cos 3\pi \operatorname{ctg}\frac{7\pi}{6}\sin 220^\circ}$
3. Израчунати: $\cos(-945^\circ)$, $\sin(\frac{38\pi}{3})$, $\operatorname{ctg} 330^\circ$, $\operatorname{tg}\frac{4\pi}{3}$
4. Ако је $\sin\alpha = \frac{1}{3}$, и $\alpha \in II$ квадранту, одредити вредности осталих тригонометријских ф-ја.

Друга група

1. Упростити израз: $\frac{\cos(\pi-\alpha)\cos(\frac{3\pi}{2}-\alpha)}{\sin(\frac{\pi}{2}+\alpha)\sin(\alpha-3\pi)}$
2. Упростити израз: $\frac{\cos(-\pi)\operatorname{tg}\frac{4\pi}{3}\sin 380^\circ}{\sin\frac{5\pi}{2}\operatorname{ctg}\frac{13\pi}{6}\cos 290^\circ}$
3. Израчунати: $\sin(-1110^\circ)$, $\cos(\frac{41\pi}{4})$, $\operatorname{tg} 300^\circ$, $\operatorname{ctg}\frac{5\pi}{6}$
4. Ако је $\cos\alpha = -\frac{1}{4}$, $\alpha \in III$ квадранту, одредити вредности осталих тригонометријских ф-ја.

Решења:

① ΥΠΟΘΕΣΗ: $\alpha = \frac{\pi}{2} - \beta$

$$(i) \frac{\cos(\pi + \alpha) \cdot \sin(\frac{3\pi}{2} + \alpha)}{\sin(\frac{\pi}{2} - \alpha) \cdot \cos(\alpha - 3\pi)} = \frac{-\cos \alpha \cdot (-\cos \alpha)}{\cos \alpha \cdot \cos(\pi - \alpha)} = \frac{\cos \alpha}{-\cos \alpha} = -1$$

$$(ii) \frac{\cos(\pi - \alpha) \cdot \cos(\frac{3\pi}{2} - \alpha)}{\sin(\frac{\pi}{2} + \alpha) \cdot \sin(\alpha - 3\pi)} = \frac{-\cos \alpha \cdot (-\sin \alpha)}{\cos \alpha \cdot (-\sin(\pi - \alpha))} = \frac{\sin \alpha}{-\sin \alpha} = -1$$

② ΥΠΟΘΕΣΗ: $\alpha = 2\beta$

$$(i) \frac{\sin(-\frac{3\pi}{2}) \cdot \operatorname{tg} \frac{7\pi}{3} \cdot \cos 770^\circ}{\cos 3\pi \cdot \operatorname{ctg} \frac{7\pi}{6} \cdot \sin 220^\circ} = \frac{-\sin \frac{3\pi}{2} \cdot \operatorname{tg}(2\pi + \frac{\pi}{3}) \cdot \cos(50^\circ + 2 \cdot 360^\circ)}{\cos(\pi + 2\pi) \cdot \operatorname{ctg}(\pi + \frac{\pi}{6}) \cdot \sin(270^\circ - 50^\circ)}$$

$$= \frac{-(-1) \cdot \operatorname{tg} \frac{\pi}{3} \cdot \cos 50^\circ}{\cos \pi \cdot \operatorname{ctg} \frac{\pi}{6} \cdot (-\cos 50^\circ)} = \frac{\cos 50^\circ}{-(-\cos 50^\circ)} = 1$$

$$(ii) \frac{\cos(-\pi) \cdot \operatorname{tg} \frac{4\pi}{3} \cdot \sin 380^\circ}{\sin \frac{5\pi}{2} \cdot \operatorname{ctg} \frac{13\pi}{6} \cdot \cos 290^\circ} = \frac{\cos \pi \cdot \operatorname{tg}(\pi + \frac{2\pi}{3}) \cdot \sin(20^\circ + 360^\circ)}{\sin(\frac{\pi}{2} + 2\pi) \cdot \operatorname{ctg}(2\pi + \frac{\pi}{6}) \cdot \cos(270^\circ + 20^\circ)}$$

$$= \frac{-1 \cdot \operatorname{tg} \frac{2\pi}{3} \cdot \sin 20^\circ}{\sin \frac{\pi}{2} \cdot \operatorname{ctg} \frac{\pi}{6} \cdot \sin 20^\circ} = -1$$

③ ΙΣΠΑΥΤΗΟΤΗΤΕΣ:

$$(i) \cos(-945^\circ) = \cos(225^\circ + 2 \cdot 360^\circ) = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\sin \frac{38\pi}{3} = \sin(\frac{2\pi}{3} + \frac{36\pi}{3}) = \sin(\pi - \frac{\pi}{3} + 12\pi) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\operatorname{ctg} 330^\circ = \operatorname{ctg}(360^\circ - 30^\circ) = -\operatorname{ctg} 30^\circ = -\sqrt{3}$$

$$\operatorname{tg} \frac{4\pi}{3} = \operatorname{tg}(\pi + \frac{\pi}{3}) = \operatorname{tg} \frac{\pi}{3} = \sqrt{3}$$

$$(ii) \sin(-1110^\circ) = -\sin(30^\circ + 3 \cdot 360^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos \frac{41\pi}{4} = \cos(\frac{\pi}{4} + \frac{40\pi}{4}) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\operatorname{tg} 300^\circ = \operatorname{tg}(360^\circ - 60^\circ) = -\operatorname{tg} 60^\circ = -\sqrt{3}$$

$$\operatorname{ctg} \frac{5\pi}{6} = \operatorname{ctg}(\pi - \frac{\pi}{6}) = -\operatorname{ctg} \frac{\pi}{6} = -\sqrt{3}$$

① Ηάνω βρεθούν οσάου τριγωνομετρικών φυνκτιών

αυο γε: (I) $\sin \alpha = \frac{1}{3}$ u α ηρυναα II κβάρηηη

(II) $\cos \alpha = -\frac{1}{4}$ u α ηρυναα III κβάρηηη

$$(I) \sin^2 \alpha + \cos^2 \alpha = 1$$

$$\frac{1}{9} + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = \frac{8}{9}$$

$$\cos \alpha = \pm \frac{2\sqrt{2}}{3}$$

$$\text{Kako } \alpha \in \text{II} \Rightarrow \cos \alpha = -\frac{2\sqrt{2}}{3}$$

$$\text{tg } \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$\text{ctg } \alpha = -2\sqrt{2}$$

$$(II) \sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \frac{1}{16} = 1$$

$$\sin^2 \alpha = \frac{15}{16}$$

$$\sin \alpha = \pm \frac{\sqrt{15}}{4}$$

$$\text{Kako } \alpha \in \text{III} \Rightarrow \sin \alpha = -\frac{\sqrt{15}}{4}$$

$$\text{tg } \alpha = \frac{\sin \alpha}{\cos \alpha} = \sqrt{15}$$

$$\text{ctg } \alpha = \frac{\sqrt{15}}{15}$$