

ПЕШЕЛТА (○ - 1. рпрна; □ - 2. рпрна):

① $a=9\text{ cm}, b=4\text{ cm}, c=13\text{ cm}, d=1\text{ cm}, P, V=?$

$h^2 = c^2 - (a-b)^2 = 13^2 - 5^2 = 144; h=12\text{ cm}$

$r_1 = a+d = 10\text{ cm}; V = V_{\text{zK}} - V_v = \frac{\pi \bar{u}}{3} (r_1^2 + r_1 r_2 + r_2^2) - r_v^2 \bar{m} \cdot H =$

$r_2 = b+d = 5\text{ cm};$

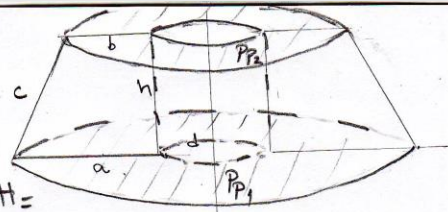
$H = h = 12\text{ cm};$

$r_v = d = 1\text{ cm};$

$\Delta = c = 13\text{ cm};$

$= \frac{\pi \bar{u}}{3} (r_1^2 + r_1 r_2 + r_2^2 - 3r_v^2) = 4\bar{m} (100 + 50 + 25 - 3) = 4 \cdot 172\bar{m} = 688\bar{m}\text{ cm}^3$

$P = P_{P_1} + P_{P_2} + M_{\text{zK}} + M_v = (r_1^2 + r_2^2 - 2r_v^2 + (r_1 + r_2) \cdot \Delta + 2r_v \cdot H) \bar{u} =$
 $= (100 + 25 - 2 + 15 \cdot 13 + 2 \cdot 12) \bar{m} = 342\bar{m}\text{ cm}^2$



① $a=20\text{ cm}, b=8\text{ cm}, c=10\text{ cm}, d=2\text{ cm}, P, V=?$

$h^2 = c^2 - \left(\frac{a-b}{2}\right)^2 = 100 - 36 = 64; h=8\text{ cm}$

$r_1 = h+d = 10\text{ cm} = r_v$

$r_2 = d = 2\text{ cm} = r_v'$

$H_{\text{zK}} = \frac{a-b}{2} = 6\text{ cm}$

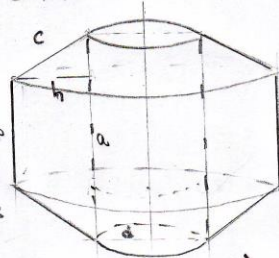
$\Delta = c = 10\text{ cm}$

$H_v = b = 8\text{ cm}$

$H_v' = a = 20\text{ cm}$

$V = 2 \cdot V_{\text{zK}} + V_v - V_v' =$
 $= \bar{u} \left[2 \cdot \frac{\pi}{3} (r_1^2 + r_1 r_2 + r_2^2) + r_1^2 H_v - r_2^2 H_v' \right] =$
 $= \bar{u} \left[2 \cdot \frac{\pi}{3} \cdot 124 + 800 - 80 \right] = 1216\bar{u}\text{ cm}^3$

$P = 2 \cdot H_{\text{zK}} + M_v + M_v' = \bar{u} (2(r_1 + r_2) \cdot \Delta + 2r_1 \cdot H_v + 2r_2 \cdot H_v') = 480\bar{u}\text{ cm}^2$



② $H=10\text{ cm}, a_1=27, b_1=25, c_1=52, O_2=72, V=?$

$O_1 = a_1 + b_1 + c_1 = 108\text{ cm};$ Коэффициент ϵ и H см и 10 cm ϵ :

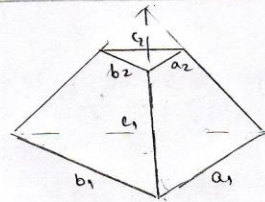
$\frac{O_1}{O_2} = \frac{108}{72} = \frac{3}{2}, n = \epsilon:$ $\frac{a_1}{a_2} = \frac{3}{2} \Rightarrow a_2 = 18\text{ cm}$

$S_1 = \frac{O_1}{2} = 54$

$S_2 = \frac{O_2}{2} = 36$

$\frac{b_1}{b_2} = \frac{3}{2} \Rightarrow b_2 = \frac{58}{3}$

$\frac{c_1}{c_2} = \frac{3}{2} \Rightarrow c_2 = \frac{104}{3}$



$B_1 = \sqrt{54 \cdot 27 \cdot 25 \cdot 2} = 54 \cdot 5 = 270\text{ cm}^2$

$B_2 = \sqrt{36 \cdot 18 \cdot \left(36 - \frac{58}{3}\right) \left(36 - \frac{104}{3}\right)} = \sqrt{18^2 \cdot 2 \cdot \frac{50}{3} \cdot \frac{4}{3}} = \frac{4 \cdot 5 \cdot 18}{3}$
 $B_2 = 120\text{ cm}^2$

$V = \frac{10}{3} (390 + \sqrt{9 \cdot 3 \cdot 4 \cdot 3 \cdot 100}) = 1900\text{ cm}^3$

② $B_1 = 36\text{ cm}^2, H = 12\text{ cm}, a_1 : a_2 = 3 : 2, V=?$

$a_1 = 3k$

$a_2 = 2k$

$B_1 = 6 \frac{a_1^2 \sqrt{3}}{4} \Rightarrow 36 = \frac{3}{2} \cdot 9k^2 \sqrt{3}; k^2 \sqrt{3} = \frac{8}{3}$

$B_2 = \frac{3}{2} a_2^2 \sqrt{3} = \frac{3}{2} 4k^2 \sqrt{3} = 6 \cdot \frac{8}{3} = 16\text{ cm}^2$

$V = \frac{12}{3} (36 + \sqrt{36 \cdot 16} + 16) = 4 \cdot 76 = 304\text{ cm}^3$



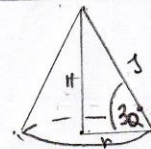
③ $\Delta = 20\text{ cm}, \varphi = 30^\circ, P, V=?$

$\sin 30^\circ = \frac{H}{\Delta} \Rightarrow H = \frac{1}{2} \cdot 20 \Rightarrow H = 10\text{ cm}$

$\cos 30^\circ = \frac{r}{\Delta} \Rightarrow r = \frac{\sqrt{3}}{2} \cdot 20 \Rightarrow r = 10\sqrt{3}\text{ cm}$

$P = \pi r (\Delta + r) = 10\sqrt{3} \pi (10\sqrt{3} + 20) = 100\sqrt{3} \pi (\sqrt{3} + 2)\text{ cm}^2$

$V = \frac{\pi r^2 \bar{m} \cdot H}{3} = \frac{100 \cdot 3 \cdot \bar{m} \cdot 10}{3} = 1000\bar{m}\text{ cm}^3$

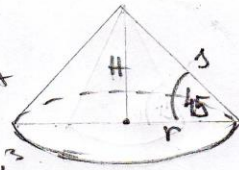


3) $OB = 18\sqrt{2} \text{ cm}$ $\varphi = 45^\circ$ $P, V = ?$

$2r\sqrt{2} = 18\sqrt{2}$ $\therefore r = 9 \text{ cm}$; Како је $\varphi = 45^\circ$: $r = H = 9 \text{ cm}$, а

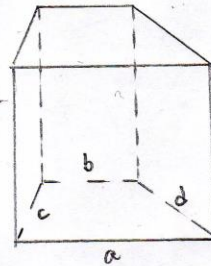
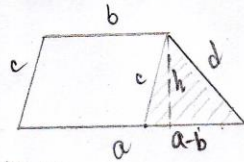
$s = r\sqrt{2}$ $\therefore s = 9\sqrt{2} \text{ cm}$:

$P = 9\sqrt{2}(9 + 9\sqrt{2}) = 81\sqrt{2}(1 + \sqrt{2}) \text{ cm}^2$; $V = \frac{9 \cdot \sqrt{2} \cdot 81}{3} = 243\sqrt{2} \text{ cm}^3$.



4) $a = 24 \text{ cm}$ $H = h$
 $b = 10 \text{ cm}$
 $c = 13 \text{ cm}$ $P, V = ?$
 $d = 15 \text{ cm}$

висина



Висина трапеца је висина троугла чије су стране $c, d, a-b$ (одељени троугао)

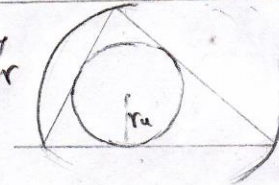
$a-b = 14$; $s = \frac{c+d+a-b}{2} = \frac{42}{2} = 21$. По Хероновом обрачу је:

$P_A = \sqrt{21 \cdot 7 \cdot 8 \cdot 6} = 42 \cdot 2 = 84 \text{ cm}^2$
 $P_A = \frac{(a-b) \cdot h}{2}$ $\left. \begin{array}{l} 84 = \frac{14 \cdot h}{2} \\ \therefore h = 12 \text{ cm} = H \end{array} \right\}$

$P = 2 \cdot B + M = 2 \cdot \frac{a+b}{2} \cdot h + (a+c+b+d) \cdot H = H(2a+2b+c+d) = 1152 \text{ cm}^2$

$V = B \cdot H = \frac{a+b}{2} \cdot h \cdot H = 17 \cdot 12^2 = 2448 \text{ cm}^3$

5) Испречићемо површину троугла, па ћемо помоћу одређених одређених попречнице описаног и уписаног круга:



5) $a = 10 \text{ cm}$, $b = 17 \text{ cm}$, $c = 21 \text{ cm}$ $S = \frac{a+b+c}{2} = 24 \text{ cm}$

$P = \sqrt{24 \cdot 14 \cdot 7 \cdot 3} = \sqrt{6 \cdot 7^3 \cdot 6 \cdot 4} = 6 \cdot 7 = 84 \text{ cm}^2$. Висина вртима једнака је $H = 2 \cdot \text{краткој}$

Висине троугла $a \cdot h_a = b \cdot h_b = c \cdot h_c = 2P \Rightarrow 10h_a = 17h_b = 21h_c = 168$, па је $H = h_c = 8$

Јав је: $P = s \cdot r_u = \frac{abc}{4r_o}$; $84 = 24 \cdot r_u \Rightarrow r_u = \frac{7}{2}$

$4 \cdot 84 = \frac{5 \cdot 10 \cdot 17 \cdot 21}{2 \cdot r_o} \Rightarrow r_o = \frac{5 \cdot 17}{8}$
 $V_o - V_u = (\frac{1}{3} \pi r_o^2 H) - (\frac{1}{3} \pi r_u^2 H) = \frac{85^2 - 28^2}{8^2} \pi \cdot 8 = \frac{57 \cdot 113}{8} \pi \cdot \text{cm}^3$

5) Задатак се на одговарајућим начин ради и за II групу, па излазнем

"Греша" резултат: $a = 25 \text{ cm}$, $b = 24 \text{ cm}$, $c = 7 \text{ cm}$

$s = 28$; $P = 84 \text{ cm}^2$; највиша висина троугла је $h_c = 24 \text{ cm} = H$.

Попречник описаног вртка: $r_o = \frac{25}{2}$
 — V — уписаног вртка: $r_u = 3$

Разлика запремина је $V_o - V_u = 19 \cdot 186 \pi \text{ cm}^3$