1) Решити квадратну једначину

$$\frac{2x+1}{x+3} - \frac{x-1}{x^2-9} = \frac{x+3}{3-x} - \frac{4+x}{3+x}$$

$$\frac{2x+1}{x+3} - \frac{x-1}{x^2-9} = -\frac{x+3}{x-3} - \frac{4+x}{3+x}$$

$$\frac{3x+5}{x+3} + \frac{x+3}{x-3} = \frac{x-1}{x^2-9} \Big/ \cdot (x-3)(x+3) \qquad x \neq \pm 3$$

$$(3x+5)(x-3) + (x+3)^2 = x-1$$

$$3x^2 - 4x - 15 + x^2 + 6x + 9 - x + 1 = 0$$

$$4x^2 + x - 5 = 0$$

$$x_{1,2} = \frac{-1 \pm 9}{8} = \begin{cases} 1 \\ -\frac{5}{4} \end{cases}$$

$$\frac{2x}{x-9} - \frac{x^2+25}{x^2-81} = \frac{5}{x+9} + \frac{5}{9-x}$$

$$\frac{2x}{x-9} - \frac{x^2+25}{x^2-81} = \frac{5}{x+9} - \frac{5}{x-9}$$

$$\frac{2x+5}{x-9} - \frac{5}{x+9} = \frac{x^2+25}{x^2-81} \Big/ (x-9)(x+9) \qquad x \neq \pm 9$$

$$(2x+5)(x+9) - 5(x-9) = x^2+25$$

$$2x^2 + 23x + 45 - 5x + 45 - x^2 - 25 = 0$$

$$x^2 + 18x + 65 = 0$$

$$x_{1,2} = \frac{-18 \pm \sqrt{324-260}}{2} = \frac{-18 \pm 8}{2} = \begin{cases} -5 \\ -13 \end{cases}$$

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2) Ако је $M = \frac{x^2-5x+4}{x^2-7x+6}$, $N = \frac{x^2-x-12}{x^2-3x-18}$

доказати да је $M - N = 0$

$$M = \frac{x^2-5x+4}{x^2-7x+6} = \frac{(x-4)\cancel{(x-1)}}{(x-6)\cancel{(x-1)}} = \frac{x-4}{x-6}$$

$$x_{1,2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}; \qquad x_{1,2} = \frac{7 \pm 5}{2} = \begin{cases} 6 \\ 1 \end{cases}$$

$$N = \frac{x^2-x-12}{x^2-3x-18} = \frac{(x-4)\cancel{(x+3)}}{(x-6)\cancel{(x+3)}} = \frac{x-4}{x-6}$$

$$x_{1,2} = \frac{1 \pm 7}{2} = \begin{cases} 4 \\ -3 \end{cases} \qquad x_{1,2} = \frac{3 \pm 9}{2} = \begin{cases} 6 \\ -3 \end{cases}$$

Т. $M = N \Rightarrow M - N = 0$

Ако је $P = \frac{a^2-12a+36}{a^2-7a+6}$, $Q = \frac{a^2-1}{a^2-5a-6}$

доказати да је $P \cdot Q = 1$.

$$P = \frac{a^2-12a+36}{a^2-7a+6} = \frac{(a-6)^2}{(a-6)(a-1)} = \frac{a-6}{a-1}$$

$$Q = \frac{a^2-1}{a^2-5x-6} = \frac{(a-1)\cancel{(a+1)}}{(a-6)\cancel{(a+1)}} = \frac{a-1}{a-6}$$

$$a_{1,2} = \frac{5 \pm 7}{2} = \begin{cases} 6 \\ -1 \end{cases}$$

$$P \cdot Q = \frac{a-6}{a-1} \cdot \frac{a-1}{a-6} = 1$$

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3) Одредити модуо комплексног броја

$$z = \frac{3-2i}{2+i} + \frac{2-i}{3+i}$$

$$z = \frac{3-2i}{2+i} \cdot \frac{2-i}{2-i} + \frac{2-i}{3+i} \cdot \frac{3-i}{3-i} = \frac{6-7i+2i^2}{4-i^2} +$$

$$+ \frac{6-5i+i^2}{9-i^2} = \frac{4-7i}{5} + \frac{5-5i}{10} = \frac{8-14i+5-5i}{10}$$

$$= \frac{13-19i}{10} \Rightarrow Re(z) = \frac{13}{10} \quad Im(z) = -\frac{19}{10}$$

$$|z|^2 = \left(\frac{13}{10}\right)^2 + \left(\frac{19}{10}\right)^2 = \frac{169+361}{100} = \frac{530}{100} = 5,3$$

$$|z| = \sqrt{5,3}$$

Одредити модуо комплексног броја

$$z = \frac{1-3i}{1+i} - \frac{i}{2+i}$$

$$z = \frac{(1-3i)(1-i)}{1-i^2} - \frac{i(2-i)}{2^2-i^2} =$$

$$= \frac{1-4i+3i^2}{2} - \frac{2i-i^2}{5} =$$

$$= \frac{-2-4i}{2} - \frac{1+2i}{5} = \frac{-\cancel{2}(1+2i)}{\cancel{2}} - \frac{1+2i}{5}$$

$$= -\frac{6}{5}(1+2i) = -\frac{6}{5} - \frac{12}{5}i$$

$$|z|^2 = \frac{36+144}{25} = \frac{180}{25} = \frac{5 \cdot 36}{25}$$

$$|z| = \frac{6\sqrt{5}}{5}$$

4) Израчунати вредност израза

$$\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} \quad \text{за} \quad x=\frac{2an}{n^2+1}$$

$$\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}\cdot\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}=$$

$$=\frac{a+\cancel{x}+2\sqrt{(a+x)(a-x)}+a-\cancel{x}}{\cancel{a}+x-\cancel{a}+x}=$$

$$=\frac{2a+2\sqrt{a^2-x^2}}{2x}=\frac{a+\sqrt{a^2-x^2}}{x}=$$

$$=\frac{a+\sqrt{a^2-\frac{4a^2n^2}{(n^2+1)^2}}}{\frac{2an}{n^2+1}}=$$

$$=\frac{\frac{a(n^2+1)+\sqrt{a^2(n^2+1)^2-4a^2n^2}}{\cancel{n^2+1}}}{\frac{2an}{\cancel{n^2+1}}}=$$

$$=\frac{a(n^2+1)+a\sqrt{n^4+2n^2+1-4n^2}}{2an}=$$

$$=\frac{n^2+1+\sqrt{(n^2-1)^2}}{2n}=\frac{n^2+\cancel{1}+n^2-\cancel{1}}{2n}=\frac{2n^2}{2n}=n$$

4) Израчунати вредност израза

$$\frac{\sqrt{a+bx}+\sqrt{a-bx}}{\sqrt{a+bx}-\sqrt{a-bx}} \quad \text{за} \quad x=\frac{2am}{b(1+m^2)}$$

$$\frac{\sqrt{a+bx}+\sqrt{a-bx}}{\sqrt{a+bx}-\sqrt{a-bx}}=\frac{\left(\sqrt{a+bx}+\sqrt{a-bx}\right)^2}{a+bx-a+bx}=$$

$$=\frac{a+\cancel{bx}+2\sqrt{a^2-b^2x^2}+a-\cancel{bx}}{2bx}=$$

$$=\frac{a+\sqrt{a^2-b^2x^2}}{bx}=\frac{a+\sqrt{a^2-\cancel{b^2}\cdot\frac{4a^2m^2}{\cancel{b^2}(1+m^2)^2}}}{\cancel{b}\cdot\frac{2am}{\cancel{b}(1+m^2)}}=$$

$$=\frac{a+a\frac{\sqrt{(1+m^2)^2-4m^2}}{1+m^2}}{\frac{2am}{1+m^2}}=$$

$$=\frac{\frac{1+m^2+\sqrt{(1-m^2)^2}}{\cancel{1+m^2}}}{\frac{2m}{\cancel{1+m^2}}}=\frac{1+\cancel{m^2}+1-\cancel{m^2}}{2m}=$$

$$=\frac{2}{2m}=\frac{1}{m}.$$

5) $$\frac{2\cdot 7^{-x}}{7^{-x}-1}-\frac{3\cdot 7^{-2x}+2\cdot 7^{-x}+1}{7^{-3x}-1}+\frac{7^{-x}+1}{7^{-2x}+7^{-x}+1}=$$

$$=\frac{2\cdot 7^{-x}}{7^{-x}-1}-\frac{3\cdot 7^{-2x}+2\cdot 7^{-x}+1}{(7^{-x}-1)(7^{-2x}+7^{-x}+1)}+\frac{7^{-x}+1}{7^{-2x}+7^{-x}+1}=$$

$$=\frac{2\cdot 7^{-x}(7^{-2x}+7^{-x}+1)-3\cdot 7^{-2x}-2\cdot 7^{-x}-1+7^{-2x}-1}{7^{-3x}-1}=$$

$$=\frac{2\cdot 7^{-3x}+\cancel{2\cdot 7^{-2x}}+\cancel{2\cdot 7^{-x}}-\cancel{3\cdot 7^{-2x}}-\cancel{2\cdot 7^{-x}}+\cancel{7^{-2x}}-2}{7^{-3x}-1}=$$

$$=\frac{2(7^{-3x}-1)}{7^{-3x}-1}=2.$$

5) $$\frac{2\cdot 5^{-x}-1}{2\cdot 5^{-x}}-\frac{1}{2\cdot 5^{-x}-4\cdot 5^{-2x}}-\frac{2\cdot 5^{-x}}{2\cdot 5^{-x}-1}.$$

$$=\frac{2\cdot 5^{-x}-1}{2\cdot 5^{-x}}+\frac{1}{2\cdot 5^{-x}(2\cdot 5^{-x}-1)}-\frac{2\cdot 5^{-x}}{2\cdot 5^{-x}-1}$$

$$=\frac{(2\cdot 5^{-x}-1)^2+1-(2\cdot 5^{-x})^2}{2\cdot 5^{-x}(2\cdot 5^{-x}-1)}=$$

$$=\frac{\cancel{4\cdot 5^{-2x}}-4\cdot 5^{-x}+1+1-\cancel{4\cdot 5^{-2x}}}{2\cdot 5^{-x}(2\cdot 5^{-x}-1)}=$$

$$=-\frac{2\cancel{(2\cdot 5^{-x}-1)}}{2\cdot 5^{-x}\cancel{(2\cdot 5^{-x}-1)}}=-\frac{1}{5^{-x}}=-5^{x}.$$