① $\int \frac{x\cos x}{\sin^3 x}\,dx = x\cos x + \frac{1}{2}\int \frac{dx}{\sin^2 x} = x\cos x - \frac{1}{2}\operatorname{ctg} x + C$

$u = x \Rightarrow du = dx$

$V = \int \frac{\cos x\,dx}{\sin^3 x} = \int t^{-3}\,dt = -\frac{1}{2}\frac{1}{\sin^2 x}$

$\sin x = t \Rightarrow \cos x\,dx = dt$

② $\int \frac{x\cos x}{\sin^2 x}\,dx = \ldots = -\frac{x}{\sin x} + \int \frac{dx}{\sin x} \overset{\text{пр. 29}}{=} -\frac{x}{\sin x} + \ln\left|\operatorname{tg}\frac{x}{2}\right| + C$

$V = \int t^{-2}\,dt = -\frac{1}{\sin x}$

$u = x$

③ $\int x \operatorname{arctg} x\,dx \quad \left(\begin{array}{l} u = \operatorname{arctg} x \\ V = \frac{x^2}{2} \end{array} \right) = \frac{x^2+1}{2}\operatorname{arctg} x - \frac{x}{2} + C$

④ $\int x \arcsin x\,dx \quad \left(\begin{array}{l} u = \arcsin x \Rightarrow du = \frac{dx}{\sqrt{1-x^2}} \\ V = \frac{x^2}{2} \end{array} \right) = \frac{x^2}{2}\arcsin x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$I_1 = \int \frac{x^2}{\sqrt{1-x^2}}\,dx = \left| \begin{array}{l} u = x \Rightarrow du = dx \\ V = \int \frac{x\,dx}{\sqrt{1-x^2}} = \ldots = -\sqrt{1-x^2} \end{array} \right| = -x\sqrt{1-x^2} + \int \sqrt{1-x^2}\,dx$

I_2

$$I_2=\int\sqrt{1-x^2}\,dx=\begin{pmatrix} x=\sin t\\ dx=\cos t\,dt\end{pmatrix}=\int\cos^2 t\,dt=\int\frac{1+\cos 2t}{2}\,dt=$$

$$=\frac12\left(t+\frac12\sin 2t\right)=\frac12\left(\arcsin x+\frac12\cdot 2\sin t\cdot\cos t\right)=$$

$$=\frac12\left(\arcsin x+x\sqrt{1-x^2}\right)$$

$$I=\frac{x^2}{2}\arcsin x-\frac12\left(-x\sqrt{1-x^2}+\frac12\arcsin x+\frac{x}{2}\sqrt{1-x^2}\right)+C=$$

$$=\frac{x^2}{2}\arcsin x+\frac14x\sqrt{1-x^2}-\frac14\arcsin x+C=\frac14\left(2x^2-1\right)\arcsin x+\frac14x\sqrt{1-x^2}+C$$

5) $$\int\frac{\ln x}{x^3}\,dx=-\frac{\ln x}{2x^2}+\frac12\int x^{-3}\,dx=-\frac{\ln x}{2x^2}+\frac12\cdot\frac{x^{-2}}{-2}+C=$$

$$\begin{pmatrix} u=\ln x\Rightarrow du=\frac1x\,dx\\ v=\int x^{-3}dx=-\frac{1}{2x^2}\end{pmatrix}\quad=-\frac{1}{2x^2}\left(\ln x+\frac12\right)+C=-\frac{1}{4x^2}\left(\ln x^2+1\right)+C$$

6) $$\int\ln^2x\,dx=x\ln^2x-2\int\ln x\,dx\overset{\text{пр7}}{=}x\ln^2x-2\cdot\left(x\ln x-x\right)+C$$

$$\begin{pmatrix} u=\ln^2x\Rightarrow du=2\ln x\cdot\frac1x\,dx\\ v=x\end{pmatrix}\quad=x\left(\ln^2x-2\ln x+2\right)+C$$

7) $$\int x^3e^{ax}\,dx=\frac{x^3e^{ax}}{a}-\frac3a\int x^2e^{ax}\,dx=$$ три узастопне парцијалне интеграције дају:

$$\begin{pmatrix} u=x^3\Rightarrow du=3x^2dx\\ v=\frac1a e^{ax}\end{pmatrix}\quad=\frac{e^{ax}}{a}\left(x^3-\frac{3x^2}{a}+\frac{6x}{a^2}-\frac{6}{a^3}\right)+C.$$

8) $$I=\int\sqrt{x^2-a^2}\,dx=x\sqrt{x^2-a^2}-\int\frac{x^2dx}{\sqrt{x^2-a^2}}=x\sqrt{x^2-a^2}-\int\frac{x^2-a^2+a^2}{\sqrt{x^2-a^2}}\,dx=$$

$$\begin{pmatrix} u=\sqrt{x^2-a^2}\to du=\frac{x\,dx}{\sqrt{x^2-a^2}}\\ v=x\end{pmatrix}=x\sqrt{x^2-a^2}-\underbrace{\int\sqrt{x^2-a^2}\,dx}_{I}-a^2\int\frac{dx}{\sqrt{x^2-a^2}}=$$

(*) $$\int\frac{dx}{\sqrt{x^2-a^2}}=\int\frac{2t}{\cancel{t^2-a^2}}\cdot\frac{\cancel{t^2-a^2}}{2t^2}\,dt=\int\frac{dt}{t}=\ln|t|+C=\ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\begin{pmatrix} \text{смена: } \sqrt{x^2-a^2}=t-x\ /^2\Rightarrow \cancel{x^2}-a^2=t^2-2tx+\cancel{x^2};\quad x=\frac{a^2+t^2}{2t}\\ dx=\frac{2t^2-a^2-t^2}{2t^2}\,dt;\quad dx=\frac{t^2-a^2}{2t^2}\,dt;\quad t-x=\frac{t^2-a^2}{2t}\end{pmatrix}$$

Биће:

$$2I\overset{*}{=}x\sqrt{x^2-a^2}-a^2\ln\left(x+\sqrt{x^2-a^2}\right)$$

$$I=\frac12\left(x\sqrt{x^2-a^2}-a^2\ln\left|x+\sqrt{x^2-a^2}\right|\right)+C$$