

Bikvadratne jednačine

1. Rešiti jednačine:

$$(a) 9x^4 - 82x^2 + 9 = 0 \quad (b) -4 + 13x^2 - 9x^4 = 0$$

$$(c) x^4 - 3x^2 + 2 = 0 \quad (d) 3x^4 - 13x^2 + 4 = 0$$

$$(e) x^4 - 3x^2 - 4 = 0 \quad (f) 4x^4 + 15x^2 - 4 = 0$$

Rešenje:

$$(a) R(j) = \left\{ \frac{1}{3}, -\frac{1}{3}, 3, -3 \right\} \quad (b) R(j) = \left\{ 1, -1, \frac{2}{3}, -\frac{2}{3} \right\} \quad (c) R(j) = \{1, -1, -\sqrt{2}, \sqrt{2}\}$$

$$(d) R(j) = \left\{ 2, -2, \sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}} \right\} \quad (e) R(j) = \{-2, 2, i, -i\} \quad (f) R(j) = \{-2i, 2i, \frac{1}{2}, -\frac{1}{2}\}$$

2. Rešiti jednačine

$$(a) \frac{4}{3} - \frac{31x^2}{9} + x^4 = 0$$

$$(b) (x^2 - 5x)^2 - 2(x^2 - 5x) = 6$$

$$(c) (x^2 - 2x + 3)^2 - 5(x^2 - 2x + 3) = 6$$

rešenje: (a) $x = \pm \frac{2}{3}, x = \pm \sqrt{3}$, (b) $x = 2 \pm \sqrt{7}, x = 3 \pm \sqrt{7}$, (c) $x = -1, x = 3, x = 1 \pm i\sqrt{3}$

3. Rešiti jednačine

$$(a) \left(\frac{x+1}{2}\right)^4 - 26\left(\frac{x+1}{2}\right)^2 + 25 = 0$$

$$(b) \left(\frac{x-3}{4}\right)^4 - 13\left(\frac{x-3}{4}\right)^2 + 36 = 0$$

$$(c) 4\left(x + \frac{1}{2}\right)^4 - 37\left(\frac{1}{2} + x\right)^2 + 9 = 0$$

$$(d) \left(\frac{x-1}{4}\right)^2 - 17\left(\frac{x-1}{2}\right)^2 + 16 = 0$$

$$(e) \left(\frac{x^2-4x+4}{9}\right)^2 - 5\left(\frac{x-2}{3}\right)^2 + 4 = 0 \quad \square$$

Rešenje:

$$(a) R(j) = \{-11, -3, 1, 9\} \quad (b) R(j) = \{-9, -5, 11, 15\} \quad (c) R(j) = \left\{-\frac{7}{2}, -1, 0, \frac{5}{2}\right\}$$

$$(d) R(j) = \{-7, -1, 3, 9\} \quad (e) R(j) = \{-4, -1, 5, 8\} \quad \square$$

Sistemi jednačina

4. Rešiti sledeće sisteme jednačina:

$$(a) 2x - y = 0, x^2 - 2x + y^2 = 0$$

$$(b) x - y + 7 = 0, x^2 + 3xy - y^2 + 2x - 5y + 64 = 0$$

$$(c) x - y = 3, x^2 + y^2 = 4$$

$$(d) x + y = 4, x^2 - 2y^2 - xy + 5x + 15 = 0$$

$$(e) xy = 5, x - 2y = 3$$

$$(f) xy = 15, x - 4y = -17$$

$$(g) x^2 + 4y^2 = 20, 3x^2 - y^2 = 47$$

$$(h) 2x^2 + y^2 = 1, 5x^2 + 2y^2 + 2 = 0$$

Rešenje:

$$(a) R(s) = \{(0, 0), (\frac{2}{5}, \frac{4}{5})\}$$

$$(b) R(s) = \{(2, 9), (-\frac{10}{3}, \frac{11}{3})\}$$

$$(c) R(s) = \{(\frac{3}{2} - \frac{i}{2}, -\frac{3}{2} - \frac{i}{2}), (\frac{3}{2} + \frac{i}{2}, -\frac{3}{2} + \frac{i}{2})\}$$

$$(d) R(s) = \{(1, 3)\}$$

$$(e) R(s) = \{(5, 1), (-2, -\frac{5}{2})\}$$

$$(f) R(s) = \{(3, 5), (-20, -\frac{3}{4})\}$$

$$(g) R(s) = \{(-4, -1), (-4, 1), (4, -1), (4, 1)\}$$

$$(h) R(s) = \{(-2i, -3), (-2i, 3), (2i, -3), (2i, 3)\}$$

Kvadratna funkcija

5. Ispitati tok i skicirati grafike sledećih funkcija

$$(a) f(x) = x^2 - 4$$

$$(b) f(x) = -2x^2 - 1$$

$$(c) f(x) = x^2 - 2x$$

$$(d) f(x) = x^2 - 4x + 3$$

$$(e) f(x) = -x^2 + 6x - 5$$

$$(f) f(x) = -\frac{1}{3}x^2 - x - \frac{3}{4}$$

$$(g) f(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{5}{4}$$

$$(h) f(x) = x^2 + 4x + 4$$

$$(i) f(x) = x^2 - 4x + 5$$

rešenje:

(a) Domen: $x \in R$, Nule: $x = \pm 2$, Presek sa y-osom: $y = -4$, Teme: $T_{\min}(0, -4)$, Kodomen: $\overline{D}_f: [-4, +\infty)$, Monotonost: $f(x) \searrow$ za $x \in (-\infty, 0)$, $f(x) \nearrow$ za $x \in (0, +\infty)$, Znak: $f(x) > 0$ za $x \in (-\infty, -2) \cup (2, +\infty)$, $f(x) < 0$ za $x \in (-2, 2)$

(b) Domen: $x \in R$, Nule: nema, Presek sa y-osom: $y = -1$, Teme: $T_{\max}(0, -1)$, Kodomen: $\overline{D}_f: (-\infty, -1]$, Monotonost: $f(x) \searrow$ za $x \in (0, +\infty)$, $f(x) \nearrow$ za $x \in (-\infty, 0)$, Znak: $f(x) < 0$ za $\forall x \in R$

(c) Domen: $x \in R$, Nule: $x = 0, x = 2$, Presek sa y-osom: $y = 0$, Teme: $T_{\min}(1, -1)$, Kodomen: $\overline{D}_f: [-1, +\infty)$, Monotonost: $f(x) \searrow$ za $x \in (-\infty, 1)$, $f(x) \nearrow$ za $x \in (1, +\infty)$, Znak: $f(x) > 0$ za $x \in (-\infty, 0) \cup (2, +\infty)$, $f(x) < 0$ za $x \in (0, 2)$

(d) Domen: $x \in R$, Nule: $x = 1, x = 3$, Presek sa y-osom: $y = 3$, Teme: $T_{\min}(2, -1)$, Kodomen: $\overline{D}_f: [-1, +\infty)$, Monotonost: $f(x) \searrow$ za $x \in (-\infty, 2)$, $f(x) \nearrow$ za $x \in (2, +\infty)$, Znak: $f(x) > 0$ za $x \in (-\infty, 1) \cup (3, +\infty)$, $f(x) < 0$ za $x \in (1, 3)$

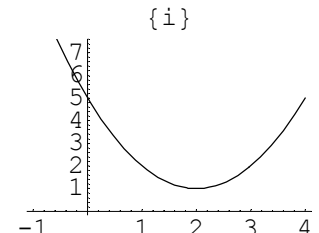
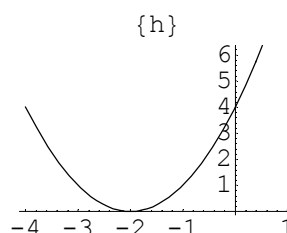
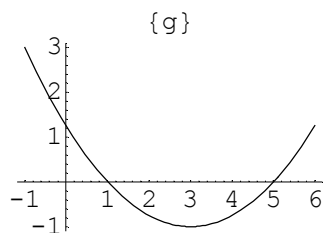
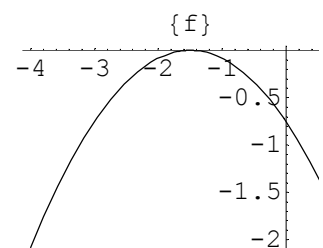
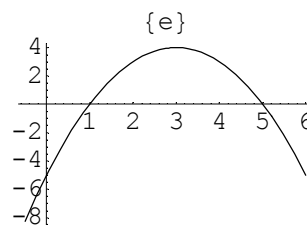
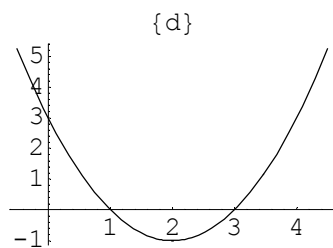
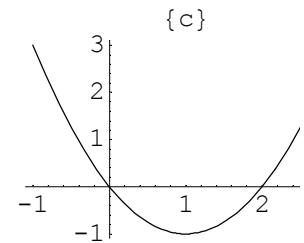
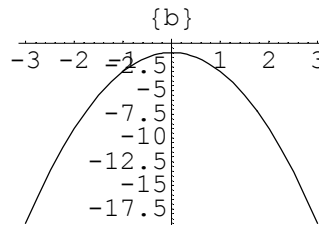
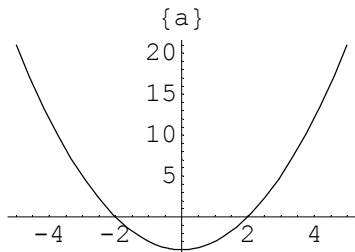
(e) Domen: $x \in R$, Nule: $x = 1, x = 5$, Presek sa y-osom: $y = -5$, Teme: $T_{\max}(3, 4)$, Kodomen: $\overline{D}_f: (-\infty, 4]$, Monotonost: $f(x) \searrow$ za $x \in (3, +\infty)$, $f(x) \nearrow$ za $x \in (-\infty, 3)$, Znak: $f(x) < 0$ za $x \in (-\infty, 1) \cup (5, +\infty)$, $f(x) > 0$ za $x \in (1, 5)$

(f) Domen: $x \in R$, Nule: $x = -\frac{3}{2}$, Presek sa y-osom: $y = -\frac{3}{4}$, Teme: $T_{\max}(-\frac{3}{2}, 0)$, Kodomen: $\overline{D}_f: (-\infty, 0]$, Monotonost: $f(x) \searrow$ za $x \in (-\frac{3}{2}, +\infty)$, $f(x) \nearrow$ za $x \in (-\infty, -\frac{3}{2})$, Znak: $f(x) < 0$ za $x \in (-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, +\infty)$

(g) Domen: $x \in R$, Nule: $x = 1, x = 5$, Presek sa y-osom: $y = \frac{5}{4}$, Teme: $T_{\min}(3, -1)$, Kodomen: $\overline{D}_f: [-1, +\infty)$, Monotonost: $f(x) \searrow$ za $x \in (-\infty, 3)$, $f(x) \nearrow$ za $x \in (3, +\infty)$, Znak: $f(x) > 0$ za $x \in (-\infty, 1) \cup (5, +\infty)$, $f(x) < 0$ za $x \in (1, 5)$

(h) Domen: $x \in \mathbb{R}$, Nule: $x = -2$, Presek sa y-osom: $y = 4$, Teme: $T_{\min}(-2, 0)$, Kodomen: $\bar{D}_f : [0, +\infty)$,
 Monotonost: $f(x) \searrow$ za $x \in (-\infty, -2)$, $f(x) \nearrow$ za $x \in (-2, +\infty)$, Znak: $f(x) > 0$ za $x \in (-\infty, -2) \cup (-2, +\infty)$

(i) Domen: $x \in \mathbb{R}$, Nule: nema, Presek sa y-osom: $y = 5$, Teme: $T_{\min}(2, 1)$, Kodomen: $\bar{D}_f : [1, +\infty)$, Monotonost: $f(x) \searrow$ za $x \in (-\infty, 2)$, $f(x) \nearrow$ za $x \in (2, +\infty)$, Znak: $f(x) > 0$ za $x \in \mathbb{R}$



Kvadratne nejednačine

6. Rešiti nejednačine

(a) $2x^2 - 3x - 2 > 0$

(b) $2x^2 + 5x > 0$

(c) $-2x^2 + 3x + 5 > 0$

(d) $x^2 - 3x - 4 \leq 0$

rešenje:

(a) $x \in \left(-\infty, -\frac{1}{2}\right) \cup (2, \infty)$, (b) $x \in \left(-\infty, -\frac{5}{2}\right) \cup (0, \infty)$

(c) $x \in \left(-1, \frac{5}{2}\right)$, (d) $x \in [-1, 4]$

7. Rešiti nejednačine:

(a) $\frac{2x^2 - 3}{2} - \frac{x^2 + 3}{4} > 0$

(b) $x^2 - 15x > 3(108 - 5x)$

rešenje:

$$(a) x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) \quad (b) x \in (-\infty, -18) \cup (18, \infty)$$

8. Rešiti nejednačinu

$$\begin{aligned} (a) x^2 - 3x > 0 \\ (b) x^2 - 7x + 10 &\leq 0 \\ (c) 2x^2 + x + 6 &\geq 0 \\ (d) -x^2 + 6x - 9 &\geq 0 \\ (e) -x^2 - 4x - 5 &> 0 \end{aligned}$$

rešenje: (a) $x \in (-\infty, 0) \cup (3, \infty)$, (b) $x \in [2, 5]$, (c) $x \in \mathbb{R}$, (d) $x = 3$, (e) nema rešenja

9. Rešiti nejednačinu

$$\begin{aligned} (a) \frac{x^2 - 4}{x + 1} < 0 \\ (b) \frac{x^2 - x - 2}{x^2 - x - 6} &\geq 0 \\ (c) \frac{-x^2 + 2x - 3}{x^2 - 4x + 3} &< 3 \\ (d) \frac{x - 2}{x^2 + 3x - 4} &\leq \frac{1}{3} \\ (e) 1 < \frac{3x^2 - 7x + 8}{x^2 + 1} &< 2 \end{aligned}$$

rešenje: (a) $x \in (-\infty, -2) \cup (-1, 2)$, (b) $x \in (-\infty, -2) \cup [-1, 2] \cup (3, \infty)$,
(c) $x \in (-\infty, 1) \cup [\frac{3}{2}, 2] \cup (3, \infty)$, (d) $x \in (-\infty, -4) \cup (1, \infty)$, (e) $x \in (1, 6)$

10. Rešiti nejednačine:

$$(a) \frac{-x^2 + 5x - 4}{x^2 - 5x + 6} \geq 0 \quad (b) \frac{x^2 + 4x - 18}{5 - x} \geq -2 \quad (c) \frac{-x^2 + 2x - 16}{x - 6} \geq 3 \quad (d) 1 < \frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$$

rešenje:

(a) $x \in [1, 2) \cup (3, 4]$, (b) $x \in (-\infty, -4] \cup [2, 5)$, (c) $x \in (-\infty, -2) \cup [1, 6)$, (d) $x \in [1, 6]$

11. Rešiti nejednačine:

$$(a) \frac{3x^2 + 6x - 9}{2x + 1} \geq x + 1 \quad (b) \frac{6}{x - 4} < 1 + x \quad (c) \left| \frac{x^2 - 3x - 4}{x^2 + x + 1} \right| < 2 \quad (d) \left| \frac{x^2 + 5x + 12}{x^2 + 9x + 12} \right| \leq 1$$

rešenje:

(a) $x \in [-5, -\frac{1}{2}) \cup (2, \infty]$, (b) $x \in (-2, 4) \cup (5, \infty)$, (c) $x \in (-\infty, -3) \cup (-2, -\frac{2}{3}) \cup (1, \infty)$,
(d) $x \in [-4, -3] \cup (0, \infty)$