

1) Требао је израчунати промене и скицати график фјне:

(I група) $f(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{5}{4}$

Домен: $D_f = \mathbb{R}$

Учне: $\frac{1}{4}x^2 - \frac{3}{2}x + \frac{5}{4} = 0 \quad | \cdot 4$
 $x^2 - 6x + 5 = 0 \quad x_{1,2} = \frac{6 \pm 1}{2} = \begin{matrix} 5 \\ 1 \end{matrix}$

A(5,0) B(1,0)

Грч: $x=0$; C(0, $\frac{5}{4}$)

Теме: $d = \frac{\frac{3}{2}}{\frac{1}{4}} = 3$

$\beta = \frac{\frac{5}{4} - \frac{9}{4}}{1} = -1 \quad T(3, 1)$

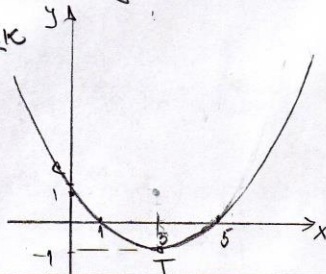
Знак: $++ \quad | \quad -- \quad | \quad ++$

$y > 0: x \in (-\infty, 1) \cup (5, +\infty)$

$y < 0: x \in (1, 5)$

Монотоност: $y \downarrow: x \in (-\infty, 3)$
 $y \uparrow: x \in (3, +\infty)$

График



(II група) $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 6$

Домен: $D_f = \mathbb{R}$

Учне: $-\frac{1}{2}x^2 + \frac{1}{2}x + 6 = 0 \quad | \cdot 2$
 $-x^2 + x + 12 = 0 \quad x_{1,2} = \frac{-1 \pm 7}{-2} = \begin{matrix} -3 \\ 4 \end{matrix}$

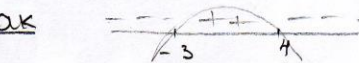
A(-3,0) B(4,0)

Грч: $x=0$; C(0,6)

Теме: $d = \frac{-\frac{1}{2}}{-1} = \frac{1}{2}$

$\beta = \frac{-12 - \frac{1}{4}}{-2} = \frac{49}{8} \quad T(\frac{1}{2}, \frac{49}{8})$

Знак



$y > 0: x \in (-3, 4)$

$y < 0: x \in (-\infty, -3) \cup (4, +\infty)$

Монотоност

$y \uparrow: x \in (-\infty, \frac{1}{2})$
 $y \downarrow: x \in (\frac{1}{2}, +\infty)$

2) (I група)

$\frac{-x^2 + 2x - 3}{x^2 - 4x + 3} \leq 3$

$\frac{-x^2 + 2x - 3 - 3x^2 + 12x - 9}{x^2 - 4x + 3} \leq 0$

$\frac{-4x^2 + 14x - 12}{x^2 - 4x + 3} \leq 0 \quad | : (-2)$

$\frac{2x^2 - 7x + 6}{x^2 - 4x + 3} \geq 0$

$x_{1,2} = \frac{7 \pm 1}{4} = \begin{matrix} 2 \\ \frac{3}{2} \end{matrix}$

$x_{3,4} = \frac{4 \pm 2}{2} = \begin{matrix} 3 \\ 1 \end{matrix}$

$x_{3,4} = \begin{matrix} 3 \\ 1 \end{matrix}$

Sign chart for $2x^2 - 7x + 6$

Sign chart for $x^2 - 4x + 3$

Sign chart for the combined inequality

$x \in (-\infty, 1) \cup [\frac{3}{2}, 2] \cup (3, +\infty)$

(II група)

$\frac{2x^2 + x - 13}{x^2 - 2x - 3} \geq 1$

$\frac{2x^2 + x - 13 - x^2 + 2x + 3}{x^2 - 2x - 3} \geq 0$

$\frac{x^2 + 3x - 10}{x^2 - 2x - 3} \geq 0$

$x_{1,2} = \frac{-3 \pm 7}{2} = \begin{matrix} 2 \\ -5 \end{matrix}$

$x_{3,4} = \frac{2 \pm 4}{2} = \begin{matrix} 3 \\ -1 \end{matrix}$

Sign chart for $x^2 + 3x - 10$

Sign chart for $x^2 - 2x - 3$

Sign chart for the combined inequality

$x \in (-\infty, -5] \cup (-1, 2] \cup (3, +\infty)$

3) (I рина)

$$\sqrt{2x+3} - \sqrt{x-1} = \sqrt{3x-8} \quad |^2$$

$$x \geq -\frac{3}{2} \wedge x \geq 1 \wedge x \geq \frac{8}{3}$$

$$R: x \in \left[\frac{8}{3}, +\infty \right)$$

$$2x+3 + x-1 + 2\sqrt{(2x+3)(x-1)} = 3x-8$$

$$-2\sqrt{(2x+3)(x-1)} = -10$$

$$\sqrt{(2x+3)(x-1)} = 5 \quad |^2$$

$$(2x+3)(x-1) = 25$$

$$2x^2 + x - 28 = 0 \quad \frac{7}{2}$$

$$x_{1,2} = \frac{-1 \pm 15}{4} = < -4 \quad \boxed{x = \frac{7}{2}}$$

(II рина)

$$\sqrt{2x-1} + \sqrt{x-1} = \sqrt{4x+5}$$

$$x \geq \frac{1}{2} \wedge x \geq 1 \wedge x \geq -\frac{5}{4} \quad R: x \in [1, +\infty)$$

$$2x-1 + x-1 + 2\sqrt{(2x-1)(x-1)} = 4x+5$$

$$2\sqrt{(2x-1)(x-1)} = x+7 \quad |^2 \quad x \geq -7 \quad \checkmark$$

$$4(2x-1)(x-1) = x^2 + 14x + 49$$

$$8x^2 - 12x + 4 = x^2 + 14x + 49$$

$$7x^2 - 26x - 45 = 0 \quad \frac{70}{14} = 5 \quad \checkmark$$

$$x_{1,2} = \frac{26 \pm 44}{14} = < -\frac{18}{14} = -\frac{9}{7}$$

$$\boxed{x = 5}$$

4) (I рина) $(5k-1)x^2 - (5k+2)x + 3k-2 = 0$

$$5k-1 \neq 0; k \neq \frac{1}{5}$$

$$D = (5k+2)^2 - 4(5k-1)(3k-2) =$$

$$= -35k^2 + 72k - 4$$

1° рел о ПД: $D = 0$:

(ФЕВШТЕ КВАДРАТН] 2, 7, 7!)

$$k_1 = 2; k_2 = \frac{2}{35}$$

$$2 \quad \begin{array}{c} + + \\ \frac{2}{35} \quad 2 \end{array}$$

2° рел о ПП: $D > 0$

$$k \in \left(\frac{2}{35}, \frac{1}{5} \right) \cup \left(\frac{1}{5}, 2 \right)$$

3° рел о КК: $D < 0$

$$k \in \left(-\infty, \frac{2}{35} \right) \cup \left(2, +\infty \right)$$

(II рина) $(2m+1)x^2 - (m+2)x + m-3 = 0$

$$m \neq -\frac{1}{2}$$

$$D = (m+2)^2 - 4(2m+1)(m-3) = -7m^2 + 24m + 16$$

1° рел о ПД: $D = 0$:

$$-7m^2 + 24m + 16 = 0; m_{1,2} = \frac{-24 \pm 32}{-14}$$

$$m_1 = -\frac{4}{7} \quad \text{или} \quad m_2 = 4$$

$$\begin{array}{c} - \quad + + + + \quad - \\ -\frac{4}{7} \quad -1/2 \quad 4 \end{array}$$

2° рел о ПП: $D > 0$:

$$m \in \left(-\frac{4}{7}, -\frac{1}{2} \right) \cup \left(-\frac{1}{2}, 4 \right)$$

3° рел о КК: $D < 0$

$$m \in \left(-\infty, -\frac{4}{7} \right) \cup \left(4, +\infty \right)$$

5) (I рина) $3x^2 + 2xy + 2y^2 + 3x - 4y = 0$

$$2x - y + 5 = 0$$

$$y = 2x + 5$$

$$3x^2 + 2x(2x+5) + 2(2x+5)^2 + 3x - 4(2x+5) = 0$$

$$3x^2 + 4x^2 + 10x + 8x^2 + 40x + 50 + 3x - 8x - 20 = 0$$

$$y = 2x + 5$$

$$15x^2 + 45x + 30 = 0 \quad | :15$$

$$y = 2x + 5$$

$$x^2 + 3x + 2 = 0$$

$$y = 2x + 5$$

$$x_{1,2} = \frac{-3 \pm 1}{2}$$

$$x_1 = -1; y_1 = 3 \quad (-1, 3)$$

$$x_2 = -2; y_2 = 1 \quad (-2, 1)$$

(II рина)

$$3x^2 + 2xy - y^2 + 6x + 4y = 3$$

$$x - 5y = -5$$

$$x = 5y - 5 = 5(y-1)$$

$$3 \cdot 25(y-1)^2 + 10(y-1) \cdot y - y^2 + 30(y-1) + 4y = 3$$

$$75y^2 - 150y + 75 + 10y^2 - 10y - y^2 + 30y - 30 + 4y - 3 = 0$$

$$x = 5(y-1)$$

$$84y^2 - 126y + 42 = 0 \quad | :42$$

$$x = 5(y-1)$$

$$2y^2 - 3y + 1 = 0$$

$$x = 5(y-1)$$

$$y_{1,2} = \frac{3 \pm 1}{4}$$

$$y_1 = 1; x_1 = 0 \quad (0, 1)$$

$$y_2 = 1/2; x_2 = -5/2 \quad \left(-\frac{5}{2}, \frac{1}{2}\right)$$