

пример 18 $\int \frac{dx}{x^2-4x+3} = \frac{1}{2} \int \frac{dx}{x-3} - \frac{1}{2} \int \frac{dx}{x-1} = \dots = \ln \sqrt{\frac{x-3}{x-1}} + C$

$\left(\frac{1}{x^2-4x+3} = \frac{1}{(x-3)(x-1)} = -\frac{x-x-3+1}{2(x-3)(x-1)} = -\frac{(x-3)-(x-1)}{2(x-3)(x-1)} \right)$
 $\left(= \frac{(x-1)-(x-3)}{2(x-3)(x-1)} = \frac{x-1}{2(x-3)(x-1)} - \frac{x-3}{2(x-3)(x-1)} = \frac{1}{2(x-3)} - \frac{1}{2(x-1)} \right)$
 3x l₁: конста t = x-3 3x l₂: конста p = x-1

пример 19 $\int \frac{2x+3}{x^2+4x+5} dx = \int \frac{2x+4-1}{x^2+4x+5} dx = \int \frac{2x+4}{x^2+4x+5} dx - \int \frac{dx}{x^2+4x+5}$
 $= \int \frac{2x+4}{x^2+4x+5} dx - \int \frac{dx}{1+(x+2)^2}$ (здесь $x^2+4x+5 = x^2+4x+4+1 = (x+2)^2+1$)
 $= \int \frac{dt}{t} - \int \frac{dp}{1+p^2} = \ln |x^2+4x+5| - \arctg(x+2) + C$
 y l₁: конста $x^2+4x+5 = t$
 $(2x+4) dx = dt$
 y l₂: конста $x+2 = p \Rightarrow dx = dp$

пример 20 $\int \frac{dx}{x^2-1} = \int \frac{dx}{(x-1)(x+1)} \stackrel{пр18}{=} \frac{1}{2} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} = \dots = \ln \sqrt{\frac{x-1}{x+1}} + C$

пример 21 $\int \frac{dx}{x^2+x-6} = \dots = \ln \sqrt[5]{\frac{x-2}{x+3}} + C$

пример 22 $\int \frac{4x+3}{x^2+x-6} = \int \frac{4x+3}{(x-2)(x+3)} dx = \int \frac{2x+2x+3}{(x-2)(x+3)} dx =$
 $= \int \frac{2(x-2)+2(x+3)+1}{(x-2)(x+3)} dx = 2 \int \frac{dx}{x+3} + 2 \int \frac{dx}{x-2} + \int \frac{dx}{(x-2)(x+3)} \stackrel{пр21}{=} \dots$
 $= \ln \left[\frac{(x+3)(x-2)^2 \sqrt{x-2}}{x+3} \right] + C$

пример 23 $\int \sin \frac{x}{2} \cos \frac{x}{3} dx = \frac{1}{2} \int \sin \left(\frac{x}{2} + \frac{x}{3} \right) dx + \frac{1}{2} \int \sin \left(\frac{x}{2} - \frac{x}{3} \right) dx =$
 $(\sin \alpha \cdot \cos \beta = \frac{1}{2} (\sin(\alpha+\beta) + \sin(\alpha-\beta))) = \dots = -3 \left(\frac{1}{5} \cos \frac{5x}{6} + 3 \cos \frac{x}{6} \right) + C$

пример 24 $\int \sin x \cdot \sin 2x dx = (\sin x = t \Rightarrow \cos x dx = dt) = \frac{2}{3} \sin^3 x + C$

пример 25 $\int \frac{dx}{x \ln x \cdot \ln(\ln x)}$ $= \int \frac{dt}{t \ln t} = \int \frac{dp}{p} = \ln p + C =$
 $(\ln x = t \Rightarrow \frac{dx}{x} = dt) \quad (\ln t = p \Rightarrow \frac{dt}{t} = dp) = \ln |\ln(\ln x)| + C$

пример 26 $\int \frac{\ln x dx}{x(\ln^2 x + 1)} = \dots = \frac{1}{2} \ln(1 + \ln^2 x) + C$

пример 27 $\int \frac{\cos^3 x}{\sin x} dx = \int \frac{\cos x (1 - \sin^2 x)}{\sin x} dx = \int \frac{\cos x}{\sin x} dx - \int \cos x \sin x dx$
 $= \ln |\sin x| - \frac{1}{2} \int \sin 2x dx = \ln |\sin x| + \frac{1}{4} \cos 2x + C$

Пример 28 $\int \frac{dx}{\sqrt{15-6x-9x^2}} = \frac{1}{4} \cdot \frac{4}{3} \int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{3} \arcsin \frac{3x+1}{4} + C$

$\left(\begin{aligned} 15-6x-9x^2 &= 16-(1+6x+9x^2) = 16-(3x+1)^2 = 16\left(1-\left(\frac{3x+1}{4}\right)^2\right) \\ \frac{3x+1}{4} &= t \rightarrow dx = \frac{4}{3} dt \end{aligned} \right)$

Пример 29 $\int \frac{dx}{\sin x} = \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{1}{2} \int \frac{dx}{\sin \frac{x}{2} \cdot \cos \frac{x}{2}} = \frac{1}{2} \int \frac{dx}{\operatorname{tg} \frac{x}{2} \cos^2 \frac{x}{2}}$

$\left(t = \operatorname{tg} \frac{x}{2} \Rightarrow dt = \frac{1}{\cos^2 \frac{x}{2}} \cdot \frac{dx}{2} \Rightarrow dx = 2 \cos^2 \frac{x}{2} dt \right)$

$= \frac{1}{2} \cdot 2 \int \frac{\cos^2 \frac{x}{2} dt}{t \cdot \cos^2 \frac{x}{2}} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C$

Пример 30 $\int \frac{dx}{\cos x} = \int \frac{dx}{\sin\left(\frac{\pi}{2}+x\right)} \stackrel{\text{пр. 29}}{=} \dots = \ln \left| \operatorname{tg} \left(\frac{\pi}{2} + \frac{x}{2} \right) \right| + C$

Пример 31 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \int \sqrt{\frac{1-x}{1+x} \cdot \frac{1+x}{1-x}} dx = \int \frac{1-x}{\sqrt{1-x^2}} dx = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x dx}{\sqrt{1-x^2}}$
 $\left(\begin{aligned} 3+1/2: 1-x^2=t \\ -x dx = \frac{dt}{2} \end{aligned} \right) = \arcsin x + \frac{1}{2} \frac{t^{1/2}}{1/2} + C = \arcsin x + \sqrt{1-x^2} + C$

Пример 32 $\int \frac{\sin x dx}{a^2 + \cos^2 x} = \frac{1}{a^2} \int \frac{\sin x dx}{1 + \left(\frac{\cos x}{a}\right)^2} = \dots = -\frac{1}{a} \operatorname{arctg} \frac{\cos x}{a} + C$

Пример 33 $\int \frac{\cos x dx}{\sqrt{a^2 - \sin^2 x}} = \frac{1}{a} \int \frac{\cos x dx}{\sqrt{1 - \left(\frac{\sin x}{a}\right)^2}} = \dots = \arcsin \left(\frac{\sin x}{a} \right) + C$

Пример 34 $\int \frac{3^x dx}{\sqrt{25-9^x}} = \frac{1}{5} \int \frac{3^x dx}{\sqrt{1 - \left(\frac{3^x}{5}\right)^2}} = \frac{1}{\ln 3} \int \frac{dt}{\sqrt{1-t^2}} = \frac{\arcsin \left(\frac{3^x}{5}\right)}{\ln 3} + C$
 $\left(\frac{3^x}{5} = t \Rightarrow 3^x \ln 3 dx = 5 dt \Rightarrow 3^x dx = \frac{5 dt}{\ln 3} \right)$

Пример 35 $\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{dx}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}} = \dots = -\arcsin \frac{1}{x} + C \quad \left[\frac{1}{x} = t \right]$

Пример 36 $\int \frac{\cos \sqrt{x} dx}{\sqrt{x}} = \dots = 2 \sin \sqrt{x} + C \quad \left[\sqrt{x} = t \right]$

Пример 37 $\int \frac{x dx}{\sin^2 2x} = \dots = -\frac{1}{4} \operatorname{ctg} 2x^2 + C \quad \left[2x^2 = t \right]$

Пример 38 $\int \cos^5 x dx = \int \cos x (1 - \sin^2 x)^2 dx = \int \cos x dx - 2 \int \cos x \sin^2 x dx + \int \cos x \sin^4 x dx = \sin x - 2 \cdot \frac{\sin^3 x}{3} + \frac{1}{5} \sin^5 x + C$
 $(\sin x = t \Rightarrow \cos x dx = dt)$