

① Треба знайти: одремити лубод, на знаменити променливу одговарати зрештошкы:

I рена  $f'(x) = \left(\frac{\operatorname{tg} x}{1-\operatorname{tg} x}\right)' = \frac{\frac{1}{\cos^2 x} (1-\operatorname{tg} x) + \frac{1}{\cos^2 x} \cdot \operatorname{tg} x}{(1-\operatorname{tg} x)^2} = \frac{1}{\cos^2 x (1-\operatorname{tg} x)^2} =$   
 $= \frac{1}{(\cos x - \sin x)^2} \Rightarrow f'\left(\frac{\pi}{3}\right) = \frac{1}{\left(\frac{1-\sqrt{3}}{2}\right)^2} = \frac{4}{4-2\sqrt{3}} = \frac{2}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2(2+\sqrt{3}).$

II рена  $f'(x) = \left(\frac{1-\sin x}{1+\cos x}\right)' = \frac{-\cos x (1+\cos x) + \sin x (1-\cos x)}{(1+\cos x)^2} = \frac{\sin x - \cos x - 1}{(1+\cos x)^2}$   
 $f'\left(\frac{\pi}{4}\right) = \frac{-1}{\left(1+\frac{\sqrt{2}}{2}\right)^2} = -\frac{4}{4+4\sqrt{2}+2} = -\frac{4}{6+4\sqrt{2}} = -\frac{2}{3+2\sqrt{2}} \cdot \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = 2(2\sqrt{2}-3).$

② I рена  $k = f'(x_0) \Rightarrow f'(x) = 4x-3 \quad k = f'(2) = 5.$   
 $x_0 = 2$

A ∈  $\Gamma \Rightarrow A(2,4)$ ;  $A(2,4) \in \Gamma$   
 $k = 5 \quad \left\{ \begin{array}{l} \Gamma - \Gamma_1 = k(x-x_0) \\ \Gamma_1 - 4 = 5(x-2) \end{array} \right.$

II рена  $5x - \Gamma - 6 = 0 \quad ;$

$k = f'(x_0) \Gamma: k = f'(2) = 4 \cdot 2 + 2 = 10$   
 $M(2,9) \in \Gamma \quad \left\{ \begin{array}{l} \Gamma - \Gamma_1 = 10(x-2) \\ \Gamma_1 - 9 = 10(x-2) \end{array} \right.$

③ Извод сложене др:

I рена  $y' = (\ln(\sqrt{x} - \sqrt{x-1}))' = \frac{1}{\sqrt{x}-\sqrt{x-1}} \cdot \left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x-1}}\right) = \frac{\sqrt{x-1}-\sqrt{x}}{\sqrt{x}-\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x}\sqrt{x-1}} =$   
 $= -\frac{1}{2\sqrt{x}\sqrt{x-1}}$

II рена  $y' = \left(\ln \sqrt{\frac{a+x}{a-x}}\right)' = \sqrt{\frac{a-x}{a+x}} \cdot \frac{1}{2\sqrt{\frac{a+x}{a-x}}} \cdot \frac{a-x+a+x}{(a-x)^2} = \frac{a-x}{a+x} \cdot \frac{a}{(a-x)^2} = \frac{a}{a^2 x^2}$

④ Знак | лубода:

I рена  $D_f = \mathbb{R}: y' = 2xe^{-x^2} - 2x \cdot x^2 e^{-x^2} = 2xe^{-x^2}(1-x^2)$   
 $y_1: x \in (-\infty, -1) \cup (0, 1)$   $y_{\max}(-1) = e^{-1} = y_{\max}(1)$   
 $y_2: x \in (-1, 0) \cup (1, +\infty)$   $y_{\min}(0) = 0.$

-	-	+	+	+	x
-	-	+	+	-	$1-x^2$
+	-	+	-	-	$y'$
-	-	+	+	-	$y$

II рена  $D_f = \mathbb{R}: y' = -2xe^x + (3-x^2)e^x = (-x^2-2x+3)e^x$

$y_1: x \in (-3, 1)$   $y_{\min}(-3) = -6e^{-3}$   
 $y_2: x \in (-\infty, -3) \cup (1, +\infty)$   $y_{\max}(1) = 2e.$

-	-	+	+	-	$y'$
-	+	+	-	-	$y$

Знаменити златок:

I  $f' = \left(x \cdot \operatorname{arctg} \frac{x}{a} - \frac{a}{2} \ln(x^2+a^2)\right)' = \operatorname{arctg} \frac{x}{a} + \frac{x}{1+\frac{x^2}{a^2}} \cdot \frac{1}{a} - \frac{a}{2} \frac{2x}{x^2+a^2} =$   
 $= \operatorname{arctg} \frac{x}{a} + \frac{a^2 x}{a^2+x^2} \cdot \frac{1}{a} - \frac{ax}{a^2+x^2} = \operatorname{arctg} \frac{x}{a}.$

II  $f' = \frac{1}{4} \cdot \frac{1-x}{1+x} \cdot \frac{1-x+1+x}{(1-x)^2} - \frac{1}{2(1+x)} = \frac{1}{4} \cdot \frac{2}{1-x^2} - \frac{1}{2(1+x)} = \frac{1+x^2-1+x^2}{2(1-x^2)} = \frac{2x^2}{2(1-x^2)} = \frac{x^2}{1-x^2}$