SVODJENJE NA I KVADRAT

Kao što smo videli do sada, trigonometrijske funkcije uglova **I kvadranta** izračunavaju se na isti način kao trigonometrijske funkcije oštrih uglova pravouglog trougla. Pokazaćemo da se preko formula, trigonometrijske funkcije proizvoljnog ugla mogu izraziti preko trigonometrijskih funkcija odgovarajućeg ugla **I kvadranta**. Taj postupak se zove ***svodjenje na I kvadrat.***

1) Iz II u I kvadrant

Važe formule za: $0 < \alpha < \frac{\pi}{2}$

$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$ odnosno $\boxed{\sin\left(90^0+\alpha\right) = \cos\alpha}$

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$ odnosno $\boxed{\cos\left(90^0+\alpha\right) = -\sin\alpha}$

$tg\left(\frac{\pi}{2}+\alpha\right) = -ctg\alpha$ odnosno $\boxed{tg\left(90^0+\alpha\right) = -ctg\alpha}$

$ctg\left(\frac{\pi}{2}+\alpha\right) = -tg\alpha$ odnosno $\boxed{ctg\left(90^0+\alpha\right) = -tg\alpha}$

$$\sin(\pi-\alpha) = \sin\alpha$$
$$\cos(\pi-\alpha) = -\cos\alpha$$
$$tg(\pi-\alpha) = -tg\alpha$$
$$ctg(\pi-\alpha) = -ctg\alpha$$

odnosno:

$$\boxed{\begin{array}{l} \sin(180^0-\alpha) = \sin\alpha \\ \cos(180^0-\alpha) = -\cos\alpha \\ tg(180^0-\alpha) = -tg\alpha \\ ctg(180^0-\alpha) = -ctg\alpha \end{array}}$$

Primeri:

a) $\sin 115^o = \sin(90^o + 25^o) = \cos 25^o$ a može i:

$\sin 115^o = \sin(180^o - 65^o) = \sin 65^o$

Naravno, već smo videli "veze'' u **I kvadrantu** i znamo da je $\cos 25^o = \sin 65^o$. Tako da možete upotrebiti bilo koju formulu iz ove dve grupe.

b) $\cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

v) $tg 141^o = tg\left(180^o - 39^o\right) = -tg 39^o$

g) $ctg 101^o = ctg\left(90^o + 11^o\right) = -tg 11^o$

2) iz III u I kvadrant

Opet imamo **dve** grupe formula:

$\sin(\pi+\alpha)=-\sin\alpha$
$\cos(\pi+\alpha)=-\cos\alpha$
$tg(\pi+\alpha)=tg\alpha$
$ctg(\pi+\alpha)=ctg\alpha$
to jest:

$$\boxed{\begin{array}{l}\sin(180^0+\alpha)=-\sin\alpha\\ \cos(180^0+\alpha)=-\cos\alpha\\ tg(180^0+\alpha)=tg\alpha\\ ctg(180^0+\alpha)=ctg\alpha\end{array}}$$

$\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$ tj. $\boxed{\sin\left(270^0-\alpha\right)=-\cos\alpha}$

$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$ tj. $\boxed{\cos\left(270^0-\alpha\right)=-\sin\alpha}$

$tg\left(\frac{3\pi}{2}-\alpha\right)=ctg\alpha$ tj. $\boxed{tg\left(270^0-\alpha\right)=ctg\alpha}$

$ctg\left(\frac{3\pi}{2}-\alpha\right)=tg\alpha$ tj $\boxed{ctg\left(270^0-\alpha\right)=tg\alpha}$

Primeri:

a) $\sin\frac{4\pi}{3}=\sin\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)=\sin\left(\pi+\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

b) $\cos 207^o=\cos\left(180^o+27^o\right)=-\cos 27^o$

v) $tg263^o=tg\left(270^o-7^o\right)=ctg7^o$

g) $ctg\frac{7\pi}{6}=ctg\left(\pi+\frac{\pi}{6}\right)=ctg\frac{\pi}{6}=\sqrt{3}$

3) <u>Iz IV u I kvadrant</u>

$\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$ tj. $\sin\left(270^0+\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha$ tj. $\cos\left(270^0+\alpha\right)=\sin\alpha$

$tg\left(\frac{3\pi}{2}+\alpha\right)=-ctg\alpha$ tj. $tg\left(270^0+\alpha\right)=-ctg\alpha$

$ctg\left(\frac{3\pi}{2}+\alpha\right)=-tg\alpha$ tj. $ctg\left(270^0+\alpha\right)=-tg\alpha$

Ako posmatramo negativan ugao $(-\alpha)$:

sin(-α) = - sinα cos(-α) = cosα tg(-α) = tgα ctg(-α) = -ctg α

Ovo nam govori da je jedino $\cos\alpha$ parna funkcija (jer “uništava’’ minus a sve ostale su neparne)

Primeri:

a) $\sin 307^o=\sin\left(270^o+37^o\right)=-\cos 37^o$

b) $\cos\left(-30^{o}\right)=\cos 30^{o}=\frac{\sqrt{3}}{2}$

v) $tg\frac{11\pi}{6}=tg\left(-\frac{\pi}{6}\right)=-tg\frac{\pi}{6}=-\frac{\sqrt{3}}{3}$

g) $ctg\left(-\frac{\pi}{3}\right)=-ctg\frac{\pi}{3}=-\frac{\sqrt{3}}{3}$

Što se tiče periodičnosti funkcija $\sin x$ i $\cos x$ već smo uočili da važi:

$$\sin(\alpha+2k\pi)=\sin\alpha \quad \text{odnosno} \quad \sin(\alpha+360^{0}\cdot k)=\sin\alpha$$
$$\cos(\alpha+2k\pi)=\cos\alpha \quad \text{odnosno} \quad \cos(\alpha+360^{0}\cdot k)=\cos\alpha$$

za ***k*** koji je **bilo koji ceo broj**.

Dakle: **osnovni period finkcija** $\sin x$ **i** $\cos x$ **je** $T=2\pi$ **odnosno** $T=360^{o}$

<u>Primeri:</u>

a) $\sin 1170^{o}=$ (oduzmimo od 1170^{o} po 360^{o} dok se ne dodje '"ispod'' 360^{o})

$$1170^{o}-360^{o}=810^{o}$$
$$810-360^{o}=450^{o}$$
$$450^{o}-360=90^{o}$$

Pa je: $\sin 1170^{o}=\sin 90^{o}=1$ ili možemo zapisati: $\sin 1170^{o}=\sin(90^{o}+3\cdot 2\pi)=\sin 90^{o}$

b) $\cos 780^{o}=$ (sličan postupak)

$$780^{o}-360^{o}=420^{o}$$
$$420^{o}-360^{o}=60^{o}$$

Pa je $\cos 780^{o}=\cos 60^{o}=\frac{1}{2}$ tj. $\cos 780^{o}=\cos(60^{o}+360^{o})=\cos 60^{o}=\frac{1}{2}$

Za tangense i kotangense važi:

$tg(\alpha + k\pi) = tg\alpha$ odnosno $tg(\alpha + k\cdot 180^0) = tg\alpha$

$ctg(\alpha + k\pi) = ctg\alpha$ odnosno $ctg(\alpha + k\cdot 180^0) = ctg\alpha$

Dakle: **osnovni period funkcija** tgx **i** $ctgx$ **je** $T = \pi$ **odnosno** $T = 180^o$

<u>Primeri:</u>

a) $tg750^o =$(odavde od 750^o oduzmemo po 180^o dok se ne " spustimo'' ispod 180^o)

$$750^o - 180^o = 570^o$$
$$570^o - 180^o = 390$$
$$390^o - 180^o = 210^o$$
$$210^o - 180^o = 30^o$$
$$tg750^o = tg30^o = \frac{\sqrt{3}}{3}$$

b) $ctg\left(-1110^o\right) = -ctg1110^o = -ctg30^o = -\sqrt{3}$ jer je $1110^o = 6\cdot 180^o + 30^o$

<u>ZADACI:</u>

1) Uprostiti izraz: $\dfrac{\sin 750^o \cdot \cos 390^o \cdot tg1140^o}{ctg405^o \cdot \sin 1860^o \cdot \cos 780^o}$

<u>Rešenja:</u> Najpre ćemo upotrebom formula sve prebaciti u **I kvadrant**!

$$\sin 750^o = \sin(30^o + 2\cdot 360^o) = \sin 30^o = \frac{1}{2}$$
$$\cos 390^o = \cos(30^o + 360^o) = \cos 30^o = \frac{\sqrt{3}}{2}$$
$$tg1140^o = tg(60^o + 6\cdot 180^o) = tg60^o = \sqrt{3}$$

$$ctg405^o = ctg(45^o + 2\cdot 180^o) = ctg45^o = 1$$
$$\sin 1860^o = \sin(60^o + 5\cdot 360^o) = \sin 60^o = \frac{\sqrt{3}}{2}$$
$$\cos 780^o = \cos(60^o + 2\cdot 360^o) = \cos 60^o = \frac{1}{2}$$

Vratimo ova rešenja u početni zadatak:

$$\frac{\sin 750^o \cdot \cos 390^o \cdot tg1140^o}{ctg405^o \cdot \sin 1860^o \cdot \cos 780^o} = \frac{\cancel{\frac{1}{2}} \cdot \cancel{\frac{\sqrt{3}}{2}} \cdot \sqrt{3}}{1 \cdot \cancel{\frac{\sqrt{3}}{2}} \cdot \cancel{\frac{1}{2}}} = \sqrt{3}$$

2) Uprosti izraz:

$$\frac{\cos\frac{17\pi}{6}\cdot\sin\frac{7\pi}{3}\cdot tg\frac{17\pi}{4}}{ctg\frac{10\pi}{3}\cdot\cos\frac{7\pi}{4}\cdot\sin\frac{8\pi}{3}}$$

Slično kao u prethodnom zadatku, sve prebacujemo u **I kvadrant.**

$$\cos\frac{17\pi}{6}=\cos\frac{17\cdot180^o}{6}=\cos510^o=\cos150^o=\cos(180^o-30^o)=-\cos30^o=-\frac{\sqrt{3}}{2}$$

$$\sin\frac{7\pi}{3}=\sin\left(\frac{\pi}{3}+\frac{6\pi}{3}\right)=\sin\left(\frac{\pi}{3}+2\pi\right)=\sin\frac{\pi}{3}=\sin60^o=\frac{\sqrt{3}}{2}$$

$$tg\frac{17\pi}{4}=tg\left(\frac{\pi}{4}+\frac{16\pi}{4}\right)=tg\left(\frac{\pi}{4}+4\pi\right)=tg\frac{\pi}{4}=tg45^o=1$$

$$ctg\frac{10\pi}{3}=ctg\left(\frac{\pi}{3}+\frac{9\pi}{3}\right)=ctg\left(\frac{\pi}{3}+3\pi\right)=ctg\frac{\pi}{3}=ctg60^o=\frac{\sqrt{3}}{3}$$

$$\cos\frac{7\pi}{4}=\cos\frac{7\cdot180^o}{4}=\cos315^o=\cos(-45^o)=\cos45^o=\frac{\sqrt{2}}{2}$$

$$\sin\frac{8\pi}{3}=\sin\left(\frac{2\pi}{3}+\frac{6\pi}{3}\right)=\sin\left(\frac{2\pi}{3}+2\pi\right)=\sin\frac{2\pi}{3}=\sin\frac{2\cdot180^o}{3}=\sin120^o=\sin(90^o+30^o)=$$

$$=\cos30^o=\frac{\sqrt{3}}{2}$$

Zamenimo ove vrednosti u zadatak:

$$\frac{\cos\frac{17\pi}{6}\cdot\sin\frac{7\pi}{3}\cdot tg\frac{17\pi}{4}}{ctg\frac{10\pi}{3}\cdot\cos\frac{7\pi}{4}\cdot\sin\frac{8\pi}{3}}=\frac{-\frac{\cancel{\sqrt{3}}}{\cancel{2}}\cdot\cancel{\frac{\sqrt{3}}{2}}\cdot1}{\frac{\cancel{\sqrt{3}}}{3}\cdot\frac{\sqrt{2}}{\cancel{2}}\cdot\cancel{\frac{\sqrt{3}}{2}}}=-\frac{3}{\sqrt{2}}=-\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{3\sqrt{2}}{2}$$

3) Dokazati indetitet:

$$\frac{\sin\alpha-2\sin(\pi-\alpha)}{\cos(\pi-\alpha)-\cos\alpha}=\frac{1}{2}tg\alpha$$

Kod indetiteta krenimo od jedne strane i transformišemo je, dok ne dodjemo do druge strane.

Važi:

$$\sin(\pi-\alpha)=\sin\alpha$$
$$\cos(\pi-\alpha)=-\cos\alpha$$

$$\frac{\sin\alpha-2\sin(\pi-\alpha)}{\cos(\pi-\alpha)-\cos\alpha}=\frac{\sin\alpha-2\sin\alpha}{-\cos\alpha-\cos\alpha}=\frac{-\sin\alpha}{-2\cos\alpha}=$$
$$=\frac{1}{2}\frac{\sin\alpha}{\cos\alpha}=\frac{1}{2}tg\alpha$$

4) Dokazati indetitet:

$$\frac{\cos\left(\frac{3\pi}{2}-\alpha\right)ctg\left(\frac{\pi}{2}+\alpha\right)\cos(-\alpha)}{\cos(2\pi+\alpha)tg(\pi-\alpha)}=-\sin\alpha$$

Važi:

$$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$$
$$ctg\left(\frac{\pi}{2}+\alpha\right)=-tg\alpha$$
$$\cos(-\alpha)=\cos\alpha$$
$$\cos(2\pi+\alpha)=\cos\alpha$$
$$tg(\pi-\alpha)=-tg\alpha$$

Pa je:

$$\frac{\cos\left(\frac{3\pi}{2}-\alpha\right)ctg\left(\frac{\pi}{2}+\alpha\right)\cos(-\alpha)}{\cos(2\pi+\alpha)tg(\pi-\alpha)}=\frac{(-\sin\alpha)\cancel{(-tg\alpha)}\cancel{(\cos\alpha)}}{\cancel{(\cos\alpha)}\cancel{(-tg\alpha)}}=-\sin\alpha$$